
Cuaderno de notas de trabajo

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Cuaderno 34

Notación

$$B = \frac{1}{\sqrt{1-v^2}} \quad ; \quad b = \frac{1}{\sqrt{1-v^2}}$$

$$D = 1 - \vec{v} \cdot \vec{v}$$

$$R = \frac{1}{r - \vec{v} \cdot \vec{r}} \quad \left(\frac{1}{r - \vec{v} \cdot \vec{r}} \right)$$

Caso Unidimensional.

Movimiento de una partícula exploradora en el campo gravitacional de un punto masa.

$$\ddot{x} = \left[\begin{aligned} & \frac{\partial \Phi}{\partial x} + \left(\frac{\partial P}{\partial x} - \frac{\partial T}{\partial t} \right) \dot{x} \\ & + \left(\frac{\partial \Phi}{\partial x} + \frac{\partial P}{\partial t} \right) \dot{x}^2 \\ & + \left(-\frac{\partial P}{\partial x} + \frac{\partial T}{\partial t} \right) \dot{x}^3 \end{aligned} \right]$$

tensor

tensor

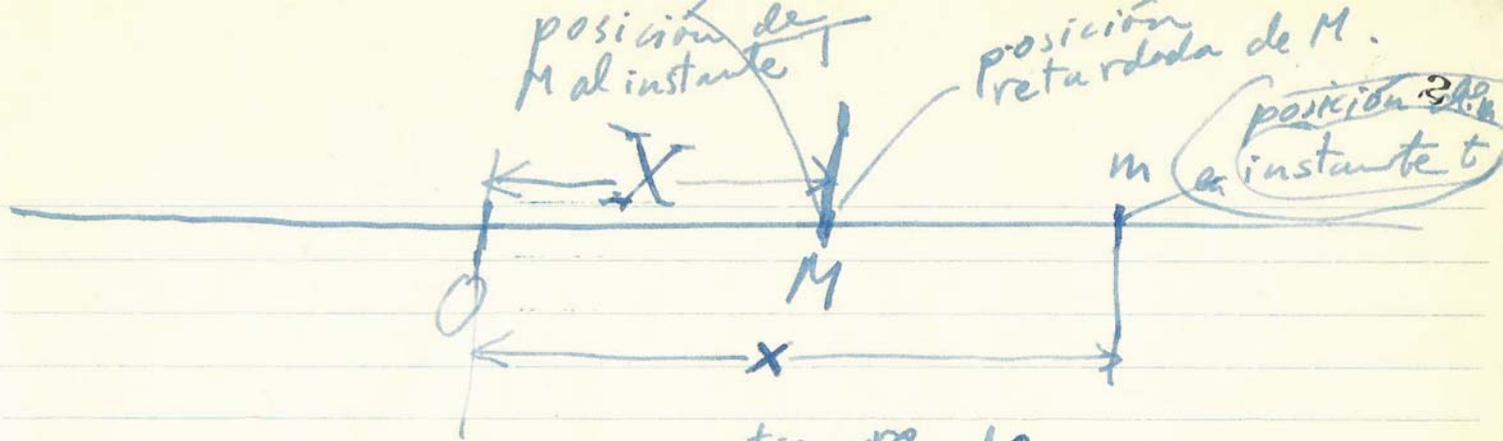
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$$\Phi = M \frac{1 + V^2}{\sqrt{1 - V^2} (r - \vec{r} \cdot \vec{V})}$$

$$\vec{P} = - \frac{2M \vec{V}}{\sqrt{1 - V^2} (r - \vec{r} \cdot \vec{V})}$$

$$T^{\alpha\beta} = \frac{2M V^\alpha V^\beta}{\sqrt{1 - V^2} (r - \vec{r} \cdot \vec{V})} + \frac{M \delta^{\alpha\beta} \sqrt{1 - V^2}}{r - \vec{r} \cdot \vec{V}}$$

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$$x - X = t - T$$

$$\frac{\partial}{\partial t}$$

$$-V \frac{\partial T}{\partial t} = 1 - \frac{\partial T}{\partial t}$$

$$\frac{\partial T}{\partial t} = \frac{1}{1 - V} > 0$$

$$\boxed{\infty > \frac{\partial T}{\partial t} > 1}$$

$$1 - \frac{\partial X}{\partial x} = - \frac{\partial T}{\partial x}$$

$$1 - V \frac{\partial T}{\partial x} = - \frac{\partial T}{\partial x}$$

$$(1 - V) \frac{\partial T}{\partial x} = -1$$

$$\boxed{\frac{\partial T}{\partial x} = - \frac{1}{1 - V}}$$

$$r = x - X$$

$$\vec{r} = x - X$$

$$V = \frac{\partial X}{\partial T}$$

$$\frac{\partial V}{\partial x} = A \left(-\frac{1}{1-V} \right) = -\frac{A}{1-V}$$

$$\boxed{\frac{\partial V}{\partial x} = -\frac{A}{1-V}}$$

$$\frac{\partial V}{\partial t} = A \cdot \frac{1}{1-V} = \frac{A}{1-V}$$

$$\boxed{\frac{\partial V}{\partial t} = \frac{A}{1-V}}$$

$$\Phi = M \frac{1+V^2}{x \sqrt{1-V^2} (1-V)}$$

$$\boxed{\Phi = M \frac{1+V^2}{x \sqrt{1-V^2} (1-V)}}$$

$$P = - \frac{2MV}{x\sqrt{1-V^2}(1-V)}$$

$$T^{\alpha\beta} = \frac{2MV^2}{x\sqrt{1-V^2}(1-V)} + \frac{M\sqrt{1-V^2}}{x(1-V)}$$

Integral de Acción del Campo Central

$$\int \frac{1}{2} e^{\frac{2M}{r}} (1+v^2) dt$$

En el campo central:

$$\hat{V} = (1, 0, 0, 0)$$

$$\hat{r} = (r, x, y, z)$$

$$\boxed{\hat{V} \cdot \hat{r} = r}$$

$$\hat{v} = \left(\frac{1}{\sqrt{1-v^2}}, \frac{v^x}{\sqrt{1-v^2}}, \frac{v^y}{\sqrt{1-v^2}}, \frac{v^z}{\sqrt{1-v^2}} \right)$$

$$(\hat{V} \cdot \hat{v}) = \frac{1}{\sqrt{1-v^2}}$$

$$\frac{1}{(\hat{V} \cdot \hat{v})^2} = 1 - v^2$$

$$1 + v^2 = 2 - \frac{1}{(\hat{V} \cdot \hat{v})^2}$$

$$d\hat{s} = (dt, dx, dy, dz)$$

$$\hat{V} \cdot d\hat{s} = dt = (\hat{V} \cdot \hat{v}) ds = dt$$

$$dt = (\hat{V}^{\hat{r}} \cdot \hat{v}) ds$$

$$\int \frac{1}{2} e^{\frac{2M}{(\hat{r} \cdot \hat{V})}} \left(2 - \frac{1}{(\hat{V} \cdot \hat{v})^2} \right) (\hat{V} \cdot \hat{v}) ds$$

$$\int \frac{1}{2} e^{\frac{2M}{(\hat{r} \cdot \hat{V})}} \left[2(\hat{V} \cdot \hat{v}) - \frac{1}{(\hat{V} \cdot \hat{v})} \right] ds$$

La integral de acción en general:

$$\hat{r} = (r, x - \bar{X}, y - \bar{Y}, z - \bar{Z})$$

$$\hat{V} = \left(\frac{1}{\sqrt{1-V^2}}, \frac{V^x}{\sqrt{1-V^2}}, \frac{V^y}{\sqrt{1-V^2}}, \frac{V^z}{\sqrt{1-V^2}} \right)$$

$$\hat{V}^{\hat{r}} = (B, BV^x, BV^y, BV^z)$$

$$(\hat{r} \cdot \hat{V}^{\hat{r}}) = B(r - \vec{r} \cdot \vec{V}) = B\mathcal{R}^{-1}$$

$$e^{\frac{2M}{(\hat{r} \cdot \hat{V})}} = e^{2MB^{-1}\mathcal{R}}$$

$$\hat{V} = (B, B\vec{V})$$

$$\hat{v} = (b, b\vec{v})$$

$$\hat{V} \cdot \hat{v} = Bb(1 - \vec{V} \cdot \vec{v})$$

$$2(\hat{V} \cdot \hat{v}) - \frac{1}{(\hat{V} \cdot \hat{v})} = \left\{ \begin{array}{l} 2Bb(1 - \vec{V} \cdot \vec{v}) + \\ \rightarrow - \frac{1}{Bb(1 - \vec{V} \cdot \vec{v})} \end{array} \right\}$$

$$b = \frac{1}{\sqrt{1 - v^2}}$$

$$ds = \sqrt{1 - v^2} dt$$

$$b ds = dt$$

$$\left[2(\hat{V} \cdot \hat{v}) - \frac{1}{(\hat{V} \cdot \hat{v})} \right] ds = \left[\begin{array}{l} 2B[1 - (\vec{V} \cdot \vec{v})] \\ - \frac{1}{Bb^2(1 - \vec{V} \cdot \vec{v})} \end{array} \right] dt$$

$$\left[2(\vec{v} \cdot \vec{v}) - \frac{1}{(\vec{v} \cdot \vec{v})} \right] ds = \left[\begin{array}{c} 2B - 2B(\vec{v} \cdot \vec{v}) \\ - \frac{1-v^2}{B(1-\vec{v} \cdot \vec{v})} \end{array} \right] dt$$

$$\int \frac{1}{2} e^{2MB^{-1}R} \left[\begin{array}{c} 2B - 2B(\vec{v} \cdot \vec{v}) \\ - \frac{1-v^2}{B(1-\vec{v} \cdot \vec{v})} \end{array} \right] ds$$

$$e^{2MB^{-1}R} \approx 1 + 2MB^{-1}R + 2M^2B^{-2}R^2$$

$$\frac{1}{1-\vec{v} \cdot \vec{v}} \approx 1 + (\vec{v} \cdot \vec{v}) + (\vec{v} \cdot \vec{v})^2$$

$$\frac{1-v^2}{1-\vec{v} \cdot \vec{v}} \approx 1 + (\vec{v} \cdot \vec{v}) + (\vec{v} \cdot \vec{v})^2 - v^2 - (\vec{v} \cdot \vec{v})v^2$$

Órdenes de magnitud en función de c^{-1}

$$\begin{array}{l|l}
 [v] = c^{-1} & [B] = 1 \\
 [V] = c^{-1} & [B^{-1}] = 1 \\
 [Q] = 1 & [t] = 1
 \end{array}$$

Aproximación Newtoniana Caso unidimensional
 ~~$\frac{d^2 x}{dt^2} = M$~~

$$\begin{aligned}
 x' &= ct \\
 [x'] &= c
 \end{aligned}$$

$$[M] = c^{-2}$$

$$\frac{d^2 x^2}{(dx')^2} = M$$

$$\left[\frac{d^2 x^2}{(dx')^2} \right] = [M] = c^{-2}$$

~~2/7~~

$$[] = \left[\begin{array}{l} \text{[0]} \quad \text{[2]} \quad \text{[0]} \quad \text{[2]} \\ 2B - 2B(\vec{v} \cdot \vec{v}) - B^{-1} - B^{-1}(\vec{v} \cdot \vec{v}) \\ - B^{-1}(\vec{v} \cdot \vec{v})^2 + B^{-1}v^2 + B^{-1}(\vec{v} \cdot \vec{v})v^2 \\ \text{[4]} \quad \text{[2]} \quad \text{[4]} \end{array} \right]$$

$$[] = \left[\begin{array}{l} 2B - B^{-1} - 2B(\vec{v} \cdot \vec{v}) - B^{-1}(\vec{v} \cdot \vec{v}) \\ + B^{-1}v^2 + B^{-1}(\vec{v} \cdot \vec{v})v^2 - B^{-1}(\vec{v} \cdot \vec{v})^2 \end{array} \right]$$

$$\frac{1}{2}e^{2MB^{-1}R} \approx \frac{1}{2} + MB^{-1}R + M^2B^{-2}R^2 + \dots$$

~~2/7~~ $B - \frac{1}{2}B^{-1}$

$$\begin{aligned} & B(\vec{v} \cdot \vec{v}) + \frac{1}{2}B^{-1}(\vec{v} \cdot \vec{v})v^2 \\ & - \frac{1}{2}B^{-1}(\vec{v} \cdot \vec{v}) - \frac{1}{2}B^{-1}(\vec{v} \cdot \vec{v})^2 \\ & + \frac{1}{2}B^{-1}v^2 - 2M(\vec{v} \cdot \vec{v})R \\ & + 2MR - MB^{-2}(\vec{v} \cdot \vec{v})R \\ & - MB^{-2}R + MB^{-2}Rv^2 \\ & + 2M^2B^{-1}R^2 \\ & - M^2B^{-3}R^2 \end{aligned}$$

$$L \approx B - \frac{1}{2} B^{-1}$$

$$- B(\vec{v} \cdot \vec{v})$$

$$- \frac{1}{2} B^{-1}(\vec{v} \cdot \vec{v})$$

$$+ \frac{1}{2} B^{-1} v^2$$

$$+ 2M \mathcal{R}$$

$$- MB^{-2} \mathcal{R}$$

$$+ \frac{1}{2} B^{-1}(\vec{v} \cdot \vec{v}) v^2$$

$$- \frac{1}{2} B^{-1}(\vec{v} \cdot \vec{v})^2$$

$$- 2M(\vec{v} \cdot \vec{v}) \mathcal{R}$$

$$- MB^{-2}(\vec{v} \cdot \vec{v}) \mathcal{R}$$

$$+ MB^{-2} \mathcal{R} v^2$$

$$+ 2M^2 B^{-1} \mathcal{R}^{-2}$$

$$- M^2 B^{-3} \mathcal{R}^2$$

1) La integral de Acción del Campo Central.

$$\mathcal{A} = \int \frac{1}{2} e^{\frac{2M}{r}} (1 + v^2) dt$$

2) Expresión de la integral de acción en forma invariante en el espacio-tiempo.

En el campo central.

$$\hat{V} = (1, 0, 0, 0)$$

$$\hat{v} = \left(\frac{1}{\sqrt{1-v^2}}, \frac{v^x}{\sqrt{1-v^2}}, \frac{v^y}{\sqrt{1-v^2}}, \frac{v^z}{\sqrt{1-v^2}} \right)$$

$$(\hat{V} \cdot \hat{v}) = \frac{1}{\sqrt{1-v^2}}$$

$$(\hat{V} \cdot \hat{v})^2 = \frac{1}{1-v^2}$$

$$\frac{1}{(\hat{V} \cdot \hat{v})^2} = 1 - v^2$$

$$\mathcal{A} = \frac{1}{(\hat{V} \cdot \hat{v})^2} = 1 + v^2$$

$$\hat{V} = (1, 0, 0, 0)$$

$$\hat{r} = (r, x, y, z)$$

$$\boxed{(\hat{V} \cdot \hat{r}) = r}$$

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2$$

$$ds^2 = (1 - v^2) dt^2$$

$$\boxed{ds = \sqrt{1 - v^2} dt}$$

$$\boxed{\sqrt{1 - v^2} = \frac{1}{(\hat{V} \cdot \hat{v})}}$$

$$ds = \frac{1}{(\hat{V} \cdot \hat{v})} dt$$

$$\boxed{dt = (\hat{V} \cdot \hat{v}) ds}$$

$$\boxed{\gamma = \int \frac{1}{2} e^{\frac{2M}{(\hat{V} \cdot \hat{r})}} \left(2 - \frac{1}{(\hat{V} \cdot \hat{v})^2} \right) (\hat{V} \cdot \hat{v}) ds}$$

$$p_{\mu} = \int \frac{1}{2} e^{\frac{2M}{(\hat{V} \cdot \hat{V})}} \left[2(\hat{V} \cdot \hat{v}) - \frac{1}{(\hat{V} \cdot \hat{v})} \right] ds$$

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Integral de Acción del Campo Central expresada en forma invariante en el espacio-tiempo.

Integral de Acción del Campo de un Punto Masa que se mueve con un cuadrivector velocidad constante \hat{V} .

Espacio físico.

$$\hat{V} = (B, B\vec{V})$$

$$B = \frac{1}{\sqrt{1-v^2}}$$

$$\neq \hat{v} = (b, b\vec{v})$$

$$b = \frac{1}{\sqrt{1-v^2}}$$

$$(\vec{V} \cdot \hat{r}) = Bb \{1 - (\vec{V} \cdot \vec{r})\}$$

$$\hat{r} = (r, x-X, y-Y, z-Z)$$

$$\hat{r} = (r, \vec{r})$$

$$(\vec{V} \cdot \hat{r}) = B [r - (\vec{V} \cdot \vec{r})]$$

$$ds = \sqrt{1 - v^2} dt$$

$$ds = b^{-1} dt$$

$$\gamma = \int \frac{1}{2} e^{2MB^{-1}R} \left[2Bb(1 - (\vec{V} \cdot \vec{r})) - \frac{1}{Bb(1 + (\vec{V} \cdot \vec{r}))} \right] b^{-1} dt$$

$$\gamma = \int \frac{1}{2} e^{2MB^{-1}R} \left[2B [1 - (\vec{V} \cdot \vec{r})] - \frac{B^{-1}b^{-2}}{[1 - (\vec{V} \cdot \vec{r})]} \right] dt$$

$$R = \frac{1}{r - (\vec{V} \cdot \vec{r})}$$

$$c^{2MB^{-1}\mathcal{R}} = \mathcal{E}$$

Programma.

1) ∇B^{-1}

2) $\nabla \mathcal{R}$

3) ∇B

4) $\nabla (\vec{v} \cdot \vec{v})$

5) $\nabla [1 - \vec{v} \cdot \vec{v}]$

6) $\nabla [1 - (\vec{v} \cdot \vec{v})]^{-1}$

7) $\frac{d}{dt} B^{-1}$

8) $\frac{d}{dt} \mathcal{R}$

9) $\frac{d}{dt} B$

10) $\frac{d}{dt} (\vec{v} \cdot \vec{v})$

11) $\frac{d}{dt} [1 - \vec{v} \cdot \vec{v}]$

12) $\frac{d}{dt} [1 - \vec{v} \cdot \vec{v}]^{-1}$

13) $\frac{d}{dt} B^{-1}$

14) $\frac{d}{dt} \vec{v}$

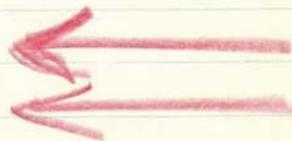
$$\textcircled{1} \quad \nabla B^{-1}$$

$$B^{-1} = \sqrt{1 - V^2}$$

$$\nabla B^{-1} = \frac{B(\vec{V} \cdot \vec{A})}{(r - (\vec{r} \cdot \vec{V}))} \vec{r}$$

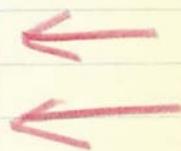
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$$\textcircled{1} \quad \nabla B^{-1} = B(\vec{V} \cdot \vec{A}) \mathcal{R} \vec{r}$$



$$\textcircled{2} \quad \nabla \mathcal{R}$$

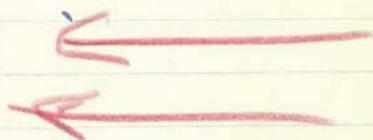
$$\textcircled{2} \quad \nabla \mathcal{R} = -B^2 \mathcal{R}^3 \vec{r} - (\vec{A} \cdot \vec{r}) \mathcal{R}^3 \vec{r} + \mathcal{R}^2 \vec{V}$$



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$$\textcircled{3} \quad \nabla B$$

$$\textcircled{3} \quad \nabla B = -B^3 (\vec{V} \cdot \vec{A}) \mathcal{R} \vec{r}$$



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grueso.

$$\textcircled{4} \nabla(\vec{V} \cdot \vec{v})$$

$$\textcircled{4} \nabla(\vec{V} \cdot \vec{v}) = -(\vec{A} \cdot \vec{v}) \mathcal{R} \vec{r}$$

$$\textcircled{5} \nabla\{1 - (\vec{V} \cdot \vec{v})\} = +(\vec{A} \cdot \vec{v}) \mathcal{R} \vec{r}$$

$$\textcircled{6} \nabla\{1 - (\vec{V} \cdot \vec{v})\}^{-1} = -\frac{(\vec{A} \cdot \vec{v}) \mathcal{R} \vec{r}}{\{1 - (\vec{V} \cdot \vec{v})\}^2}$$

Como en nuestro caso $\vec{V} = \text{const.}$

$$\gamma \vec{A} = \vec{0},$$

$$\textcircled{1} \nabla B^{-1} = 0$$

$$\textcircled{2} \nabla \mathcal{R} = -B^2 \mathcal{R}^3 \vec{r} + \mathcal{R}^2 \vec{V}$$

$$\textcircled{3} \nabla B = 0$$

$$\textcircled{4} \nabla(\vec{V} \cdot \vec{v}) = 0$$

$$\textcircled{5} \nabla\{1 - (\vec{V} \cdot \vec{v})\} = 0$$

$$\textcircled{6} \nabla\{1 - (\vec{V} \cdot \vec{v})\}^{-1} = 0$$

¡ojo!

$$\vec{V} = \text{const}; \quad \vec{A} = 0$$

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$$\textcircled{7} \frac{d}{dt} B^{-1} = 0$$

$$\textcircled{8} \boxed{\frac{d}{dt} \mathcal{R}} \quad \vec{V} = \text{const.}$$

$$\frac{d}{dt} \mathcal{R} = \frac{\partial}{\partial t} \mathcal{R} + (\vec{v} \cdot \nabla) \mathcal{R}$$

$$\frac{\partial}{\partial t} \mathcal{R} = (\vec{v} \cdot \vec{r}) \mathcal{R}^3 + B^{-2} r \mathcal{R}^3 - r \mathcal{R}^3$$

$$\vec{v} \cdot \nabla \mathcal{R} = -B^{-2} (\vec{r} \cdot \vec{v}) \mathcal{R}^3 + (\vec{V} \cdot \vec{v}) \mathcal{R}^2$$

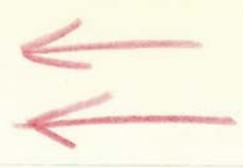
$$\frac{d}{dt} \mathcal{R} = \left[(\vec{V} \cdot \vec{v}) \mathcal{R}^2 - r \mathcal{R}^3 + B^{-2} r \mathcal{R}^3 + (\vec{V} \cdot \vec{r}) \mathcal{R}^3 - B^{-2} (\vec{r} \cdot \vec{v}) \mathcal{R}^3 \right]$$

$$\boxed{(\vec{V} \cdot \vec{r}) = r - \mathcal{R}^2}$$

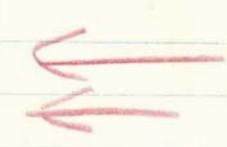
$$\textcircled{9} \frac{d}{dt} B = 0$$

$$\frac{d}{dt} \mathcal{R} = \left[-\mathcal{R}^2 + (\vec{V} \cdot \vec{v}) \mathcal{R}^2 + B^{-2} r \mathcal{R}^3 - B^{-2} (\vec{r} \cdot \vec{v}) \mathcal{R}^3 \right]$$

$$\textcircled{10} \frac{d}{dt}(\vec{v} \cdot \vec{v}) = \vec{v} \cdot \vec{a}$$



$$\textcircled{11} \frac{d}{dt}[1 - (\vec{v} \cdot \vec{v})] = -(\vec{v} \cdot \vec{a})$$



$$\textcircled{12} \frac{d}{dt}[1 - (\vec{v} \cdot \vec{v})]^{-1} = + \frac{\vec{v} \cdot \vec{a}}{[1 - (\vec{v} \cdot \vec{v})]^2}$$



$$\textcircled{13} \frac{d}{dt} B^{-1} = 0 \quad \textcircled{14} \frac{d}{dt} \vec{v} = \vec{0}$$



Ver página 214 del cuaderno grueso.

$$\begin{aligned}
 & (\vec{v} \cdot \vec{a}) = \quad (r - r^{-1}) \\
 & M R^3 [-2B^{-1} + B^{-3} b^{-2} + 4B^{-1}(\vec{v} \cdot \vec{v}) - 2B^{-1}(\vec{v} \cdot \vec{v})^2] (\vec{v} \cdot \vec{r}) \\
 & + \left\{ M R^2 [-B^{-1} b^{-2}] V^2 \right\}^{(1-B^{-2})} \\
 & + \left\{ + M R^3 [2B^{-1} r - 2B^{-1}(\vec{r} \cdot \vec{v}) - 2B^{-1}(\vec{v} \cdot \vec{v}) r + 2B^{-1}(\vec{v} \cdot \vec{v})(\vec{r} \cdot \vec{v})] V^2 \right\}^{(1-B^2)}
 \end{aligned}$$



$$+MR^2 [B^{-1} b^{-2}] (\vec{V} \cdot \vec{v})$$

$$+MR^3 \left[\begin{array}{l} -B^{-2} r b^{-2} + 2B^{-1} (\vec{r} \cdot \vec{v}) - 2B^{-1} (\vec{V} \cdot \vec{v}) r \\ -2B^{-1} (\vec{V} \cdot \vec{v}) (\vec{r} \cdot \vec{v}) + 2B^{-1} (\vec{V} \cdot \vec{v})^2 r \end{array} \right] (\vec{V} \cdot \vec{v})$$

~~$$+MR^2 [B^{-1} b^{-2}] (\vec{V} \cdot \vec{v})$$~~

$$R = \frac{1}{r - (\vec{V} \cdot \vec{r})}$$

$$R^{-1} = [r - (\vec{V} \cdot \vec{r})]$$

$$(\vec{V} \cdot \vec{r}) = r - R^{-1}$$

$$B^{-2} = 1 - V^2$$

$$V^2 = 1 - B^{-2}$$

$$\begin{aligned}
 & [-2B^{-1} + B^{-3}b^{-2} + 4B^{-1}(\vec{V} \cdot \vec{v}) - 2B^{-1}(\vec{V} \cdot \vec{v})^2] MR^3(r - \mathcal{R}^{-1}) \\
 &= MR^3 \left[\begin{array}{l} -2B^{-1}r + B^{-3}b^{-2}r + 4B^{-1}(\vec{V} \cdot \vec{v})r \\ -2B^{-1}(\vec{V} \cdot \vec{v})^2r \end{array} \right] \\
 &+ MR^{+2} \left[\begin{array}{l} -2B^{-1} + B^{-3}b^{-2} + 4B^{-1}(\vec{V} \cdot \vec{v}) \\ -2B^{-1}(\vec{V} \cdot \vec{v})^2 \end{array} \right]
 \end{aligned}$$

$$MR^2[-B^{-1}b^{-2}]V^2 =$$

$$MR^2[-B^{-1}b^{-2}][1 - B^{-2}] =$$

$$MR^2[-B^{-1}b^{-2} + B^{-3}b^{-2}]$$

$$MR^3 \left[\begin{array}{l} -2B^{-1}r - 2B^{-1}(\vec{r} \cdot \vec{v}) - 2B^{-1}(\vec{V} \cdot \vec{v})r \\ + 2B^{-1}(\vec{V} \cdot \vec{v})(\vec{r} \cdot \vec{v}) \end{array} \right] (1 - B^{-2})$$

$$MR^3 \left[\begin{array}{l} -2B^{-1}r - 2B^{-1}(\vec{r} \cdot \vec{v}) - 2B^{-1}(\vec{V} \cdot \vec{v})r \\ + 2B^{-1}(\vec{V} \cdot \vec{v})(\vec{r} \cdot \vec{v}) + 2B^{-3}r + 2B^{-3}(\vec{r} \cdot \vec{v}) \\ + 2B^{-3}(\vec{V} \cdot \vec{v})r - 2B^{-3}(\vec{V} \cdot \vec{v})(\vec{r} \cdot \vec{v}) \end{array} \right]$$

$$(\vec{V} \cdot \vec{a}) =$$

$$\begin{aligned}
& \frac{M R^2}{+ 2 B^{-1}} \checkmark \\
& \cancel{+ B^{-3} b^{-2}} \checkmark \\
& \cancel{+ 4 B^{-1} (\vec{V} \cdot \vec{v})} \checkmark \\
& \cancel{+ 2 B^{-1} (\vec{V} \cdot \vec{v})^2} \checkmark \\
& \cancel{- B^{-1} b^2} \checkmark \\
& \cancel{+ B^{-3} b^2} \checkmark \\
& \cancel{+ B^{-1} b^{-2} (\vec{V} \cdot \vec{v})} \checkmark
\end{aligned}$$

Comprobado
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$$\begin{aligned}
& + \frac{M R^3}{- 2 B^{-1} r} \checkmark \\
& + B^{-3} b^{-2} r \checkmark \\
& + 4 B^{-1} (\vec{V} \cdot \vec{v}) r \checkmark \\
& - 2 B^{-1} (\vec{V} \cdot \vec{v})^2 r \times \\
& \rightarrow + 2 B^{-1} r \checkmark \\
& - 2 B^{-1} (\vec{r} \cdot \vec{v}) \checkmark \\
& - 2 B^{-1} (\vec{V} \cdot \vec{v}) r \checkmark \\
& + 2 B^{-1} (\vec{V} \cdot \vec{v}) (\vec{r} \cdot \vec{v}) \checkmark \\
& \rightarrow \cancel{+ 2 B^{-3} r} \checkmark \\
& \rightarrow + 2 B^{-3} (\vec{r} \cdot \vec{v}) \checkmark \\
& + 2 B^{-3} (\vec{V} \cdot \vec{v}) r \checkmark \\
& - 2 B^{-3} (\vec{V} \cdot \vec{v}) (\vec{r} \cdot \vec{v}) \checkmark \\
& - B^{-3} (\vec{V} \cdot \vec{v}) r b^{-2} \checkmark \\
& + 2 B^{-1} (\vec{V} \cdot \vec{v}) (\vec{r} \cdot \vec{v}) \checkmark \\
& - 2 B^{-1} (\vec{V} \cdot \vec{v})^2 r \times \\
& - 2 B^{-1} (\vec{V} \cdot \vec{v})^2 (\vec{r} \cdot \vec{v}) \checkmark \\
& + 2 B^{-1} (\vec{V} \cdot \vec{v})^3 r \checkmark
\end{aligned}$$

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$$\frac{\partial \mathcal{L}}{\partial \vec{v}} = \frac{1}{2} e^{2MB^{-1}\mathcal{R}} \left[\begin{aligned} & -2B\vec{v} - \frac{B^{-1}b^{-2}}{[1-(\vec{v}\cdot\vec{v})]^2} \vec{v} \\ & + \frac{2B^{-1}\vec{v}}{[1-(\vec{v}\cdot\vec{v})]} \end{aligned} \right]$$

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \vec{v}} = MB^{-1} e^{2MB^{-1}\mathcal{R}} \left[\begin{aligned} & -\mathcal{R}^2 + (\vec{v}\cdot\vec{v})\mathcal{R}^2 \\ & + B^{-2}r\mathcal{R}^3 - B^{-2}(\vec{v}\cdot\vec{v})\mathcal{R}^3 \end{aligned} \right]$$

$$\left[\begin{aligned} & -2B\vec{v} - \frac{B^{-1}b^{-2}}{[1-(\vec{v}\cdot\vec{v})]^2} \vec{v} \\ & + \frac{2B^{-1}\vec{v}}{[1-(\vec{v}\cdot\vec{v})]} \end{aligned} \right]$$

$$+ \frac{1}{2} e^{2MB^{-1}\mathcal{R}} \left[\begin{aligned} & + \frac{2B^{-1}(\vec{v}\cdot\vec{a})}{[1-(\vec{v}\cdot\vec{v})]^2} \vec{v} \\ & + \frac{2B^{-1}\vec{a}}{[1-(\vec{v}\cdot\vec{v})]} \\ & + \frac{2B^{-1}\vec{v}(\vec{v}\cdot\vec{a})}{[1-(\vec{v}\cdot\vec{v})]^2} \end{aligned} \right]$$

$$- \frac{B^{-1}b^{-2}(\vec{v}\cdot\vec{a})}{[1-(\vec{v}\cdot\vec{v})]^3} \vec{v}$$

$$\nabla \mathcal{L} = MB^{-1} e^{2MB^{-1} \mathcal{R}} \left[-B^2 \mathcal{R}^3 \vec{r} + \mathcal{R}^2 \vec{V} \right]^*$$

$$\rightarrow \left[2B[1 - (\vec{V} \cdot \vec{v})] - \frac{B^{-1} b^{-2}}{[1 - (\vec{V} \cdot \vec{v})]} \right]$$

$$(\vec{v} \cdot \vec{a}) =$$

$$M \mathcal{R}^2$$

$$-B^{-1} (\vec{V} \cdot \vec{v}) b^{-2}$$

$$+ B^{-1} b^{-2}$$

$$\rightarrow B^{-1} b^{-4}$$

$$+ M \mathcal{R}^3$$

$$- 2B^{-1} (\vec{r} \cdot \vec{v})$$

$$+ B^{-3} (\vec{r} \cdot \vec{v}) b^{-2}$$

$$+ 4B^{-1} (\vec{V} \cdot \vec{v}) (\vec{r} \cdot \vec{v}) \circ$$

$$- 2B^{-1} (\vec{V} \cdot \vec{v})^2 (\vec{r} \cdot \vec{v})$$

$$+ 2B^{-1} (\vec{V} \cdot \vec{v}) r$$

$$- 2B^{-1} (\vec{V} \cdot \vec{v}) (\vec{r} \cdot \vec{v}) \circ$$

$$- 2B^{-1} (\vec{V} \cdot \vec{v})^2 r$$

$$+ 2B^{-1} (\vec{V} \cdot \vec{v})^2 (\vec{r} \cdot \vec{v})$$

$$- B^{-3} r b^{-2} \checkmark$$

$$+ 2B^{-1} (\vec{r} \cdot \vec{v})$$

$$- 2B^{-1} (\vec{V} \cdot \vec{v}) r$$

$$- 2B^{-1} (\vec{V} \cdot \vec{v}) (\vec{r} \cdot \vec{v})$$

$$+ 2B^{-1} (\vec{V} \cdot \vec{v})^2 r$$

$$+ B^{-3} r b^{-4} \checkmark$$

$$\rightarrow 2B^{-1} (\vec{r} \cdot \vec{v}) b^{-2}$$

$$+ 2B^{-1} (\vec{V} \cdot \vec{v}) r b^{-2}$$

$$+ 2B^{-1} (\vec{V} \cdot \vec{v}) (\vec{r} \cdot \vec{v}) b^{-2}$$

$$- 2B^{-1} (\vec{V} \cdot \vec{v})^2 r b^{-2}$$

$$(\vec{v} \cdot \vec{a})$$

$\underline{MR^2}$	+	MR^3
$-B^{-1}(\vec{v} \cdot \vec{v}) b^{-2}$ ✓	·	$-B^{-3} r b^{-2}$
$+ B^{-1} b^{-2}$ ✓		$+ B^{-3} r b^{-4}$
$- B^{-1} b^{-4}$ ✓		$- 2B^{-1}(\vec{r} \cdot \vec{v}) b^{-2}$ ←
		$+ 2B^{-1}(\vec{v} \cdot \vec{v}) r b^{-2}$ ←
		$+ 2B^{-1}(\vec{v} \cdot \vec{v})(\vec{r} \cdot \vec{v}) b^{-2}$ ←
		$- 2B^{-1}(\vec{v} \cdot \vec{v})^2 r b^{-2}$ ←
		$\oplus B^{-3}(\vec{r} \cdot \vec{v}) b^{-2}$

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Términos en \vec{r}

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$$\frac{MB^{-4} \mathcal{R}^3 b^{-2}}{[1 - (\vec{v} \cdot \vec{v})]} - 2MB^2 \mathcal{R}^3 [1 - \vec{v} \cdot \vec{v}]$$

$$- 2MB^2 \mathcal{R}^3 [1 - (\vec{v} \cdot \vec{v})] + \frac{M \mathcal{R}^3 b^{-2}}{[1 - \vec{v} \cdot \vec{v}]}$$

$$\mathcal{L} = \int \frac{1}{2} e^{2MB^{-1}\mathcal{R}} \left[2B [1 - (\vec{V} \cdot \vec{v})] - \frac{B^{-1}b^{-2}}{[1 - (\vec{V} \cdot \vec{v})]} \right] dt$$

$$\vec{V} = \text{const.}$$

$$B = \frac{1}{\sqrt{1 - v^2}} = \text{const.}$$

$$\mathcal{L} = \frac{1}{2} e^{2MB^{-1}\mathcal{R}} \left[2B [1 - (\vec{V} \cdot \vec{v})] - \frac{B^{-1}b^{-2}}{[1 - (\vec{V} \cdot \vec{v})]} \right]$$

$$\frac{\partial \mathcal{L}}{\partial \vec{v}} = \frac{1}{2} e^{2MB^{-1}\mathcal{R}} \left[2B \vec{V} - \frac{B^{-1}b^{-2}}{[1 - (\vec{V} \cdot \vec{v})]^2} \vec{V} + \frac{2B^{-1}\vec{v}}{[1 - (\vec{V} \cdot \vec{v})]} \right]$$

$$\frac{\partial \mathcal{L}}{\partial \vec{v}} = \frac{1}{2} e^{2MB^{-1}\mathcal{R}} \left[-2B \vec{V} - \frac{B^{-1}b^{-2}}{[1 - (\vec{V} \cdot \vec{v})]^2} \vec{V} + \frac{2B^{-1}}{[1 - (\vec{V} \cdot \vec{v})]} \vec{v} \right]$$

$$\frac{d}{dt} \mathcal{R} = \frac{\partial \mathcal{R}}{\partial t} + (\vec{v} \cdot \nabla) \mathcal{R}$$

$$\frac{d}{dt} \mathcal{R} = \left[\begin{aligned} &(\vec{v} \cdot \vec{r}) \mathcal{R}^3 + B^{-2} \mathcal{R}^3 r - r \mathcal{R}^3 \\ &- B^{-2} (\vec{r} \cdot \vec{v}) \mathcal{R}^3 + (\vec{v} \cdot \vec{v}) \mathcal{R}^2 \end{aligned} \right]$$

$$\mathcal{R}^{-1} = r - (\vec{r} \cdot \vec{v})$$

$$(\vec{v} \cdot \vec{r}) = r - \mathcal{R}^{-1}$$

$$\frac{d}{dt} \mathcal{R} = \left[\begin{aligned} &\cancel{r - \mathcal{R}^{-1}} \mathcal{R}^3 + B^{-2} \mathcal{R}^3 r - \cancel{r \mathcal{R}^3} \\ &- B^{-2} (\vec{r} \cdot \vec{v}) \mathcal{R}^3 + (\vec{v} \cdot \vec{v}) \mathcal{R}^2 \end{aligned} \right]$$

$$\frac{d}{dt} \mathcal{R} = \left[\begin{aligned} &- \mathcal{R}^2 + (\vec{v} \cdot \vec{v}) \mathcal{R}^2 + B^{-2} r \mathcal{R}^3 \\ &- B^{-2} (\vec{r} \cdot \vec{v}) \mathcal{R}^3 \end{aligned} \right]$$

$\vec{v} = \text{const}$

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$$e^{-2MB^{-1}R} \frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \vec{v}} \right) =$$

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$$MB^{-1} \begin{bmatrix} -R^2 \\ + (\vec{v} \cdot \vec{v}) R^2 \\ + B^{-2} R^3 \\ - B^{-2} (\vec{v} \cdot \vec{v}) R^3 \end{bmatrix} \cdot \begin{bmatrix} -2B\vec{v} \\ - \frac{B^{-1}b^{-2}}{\{1 - \vec{v} \cdot \vec{v}\}^2} \vec{v} \\ + \frac{2B^{-1}}{\{1 - (\vec{v} \cdot \vec{v})\}} \vec{v} \end{bmatrix}$$

$$+ \frac{B^{-1}}{\{1 - (\vec{v} \cdot \vec{v})\}} \vec{a} + \frac{B^{-1}(\vec{v} \cdot \vec{a})}{\{1 - (\vec{v} \cdot \vec{v})\}^2} \vec{v}$$

$$- \frac{B^{-1}b^{-2}(\vec{v} \cdot \vec{a})}{\{1 - (\vec{v} \cdot \vec{v})\}^3} \vec{v}$$

$$+ \frac{B^{-1}(\vec{v} \cdot \vec{a})}{\{1 - (\vec{v} \cdot \vec{v})\}^2} \vec{v}$$

$$e^{-2MB^{-1}R} \frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \vec{v}} \right) =$$

$$\frac{B^{-1}}{\{1 - (\vec{v} \cdot \vec{v})\}} \vec{a} +$$

$$+ \frac{B^{-1} (\vec{v} \cdot \vec{a})}{\{1 - (\vec{v} \cdot \vec{v})\}^2}$$

$$- \frac{2MB^{-2}R^2}{\{1 - (\vec{v} \cdot \vec{v})\}} \checkmark$$

$$+ \frac{2MB^{-2} (\vec{v} \cdot \vec{v}) R^2}{\{1 - (\vec{v} \cdot \vec{v})\}} \checkmark$$

$$+ \frac{2MB^{-4} R^3}{\{1 - (\vec{v} \cdot \vec{v})\}} \uparrow$$

$$- \frac{2MB^{-4} (\vec{v} \cdot \vec{v}) R^3}{\{1 - (\vec{v} \cdot \vec{v})\}} \uparrow$$

~~#~~

$$- \frac{\beta^{-1} b^{-2} (\vec{V} \cdot \vec{a})}{\{1 - (\vec{V} \cdot \vec{v})\}^3} \vec{V}$$

$$+ \frac{\beta^{-1} (\vec{v} \cdot \vec{a})}{\{1 - (\vec{V} \cdot \vec{v})\}^2} \vec{V}$$

$$+ \frac{M \beta^{-2} b^{-2} \mathcal{R}^2}{\{1 - (\vec{V} \cdot \vec{v})\}^2} \vec{V}$$

$$- \frac{M \beta^{-2} b^{-2} (\vec{V} \cdot \vec{v}) \mathcal{R}^2}{\{1 - (\vec{V} \cdot \vec{v})\}^2} \vec{V}$$

$$\left. \begin{matrix} + \frac{M \beta^{-2} b^{-2} \mathcal{R}^2}{\{1 - (\vec{V} \cdot \vec{v})\}^2} \vec{V} \\ - \frac{M \beta^{-2} b^{-2} (\vec{V} \cdot \vec{v}) \mathcal{R}^2}{\{1 - (\vec{V} \cdot \vec{v})\}^2} \vec{V} \end{matrix} \right\} \frac{M \beta^{-2} b^{-2} \mathcal{R}^2}{\{1 - (\vec{V} \cdot \vec{v})\}} \vec{V}$$

$$- \frac{M \beta^{-1} r b^{-2} \mathcal{R}^3}{\{1 - (\vec{V} \cdot \vec{v})\}^2} \vec{V}$$

$$+ \frac{M \beta^{-1} (\vec{r} \cdot \vec{v}) b \mathcal{R}^3}{\{1 - (\vec{V} \cdot \vec{v})\}^2} \vec{V}$$

$$+ 2M \mathcal{R}^2 \vec{V}$$

$$- 2M (\vec{V} \cdot \vec{v}) \mathcal{R}^2 \vec{V}$$

$$- 2M \beta^{-2} r \mathcal{R}^3 \vec{V}$$

$$+ 2M \beta^{-2} (\vec{r} \cdot \vec{v}) \mathcal{R}^3 \vec{V}$$



$$\nabla \mathcal{L} = MB^{-1} \nabla \mathcal{R} e^{2MB^{-1}\mathcal{R}} \left[2B \{1 - (\vec{V} \cdot \vec{V})\} - \frac{B^{-1}b^{-2}}{\{1 - (\vec{V} \cdot \vec{V})\}} \right]$$

$$\nabla \mathcal{L} = e^{2MB^{-1}\mathcal{R}} \left[2M \{1 - (\vec{V} \cdot \vec{V})\} - \frac{MB^{-2}b^{-2}}{\{1 - (\vec{V} \cdot \vec{V})\}} \right] \nabla \mathcal{R}$$

$$\nabla \mathcal{R} = -B^{-2} \mathcal{R}^3 \vec{r} + \mathcal{R}^2 \vec{V}$$

$$-e^{-2MB^{-1}\mathcal{R}} \nabla \mathcal{L} =$$

$$\left[+2MB^{-2} \{1 - (\vec{V} \cdot \vec{V})\} \mathcal{R}^3 \right]$$

$$\left[- \frac{MB^{-4}b^{-2} \mathcal{R}^3}{\{1 - (\vec{V} \cdot \vec{V})\}} \right]$$

$\vec{r} +$

$$\left[-2M \{1 - (\vec{V} \cdot \vec{V})\} \mathcal{R}^2 \right]$$

$$\left[+ \frac{MB^{-2}b^{-2} \mathcal{R}^2}{\{1 - (\vec{V} \cdot \vec{V})\}} \right]$$

\vec{V}

Términos en \vec{r}

$$\frac{MB^{-1} \mathcal{R}^3}{\{1 - (\vec{v} \cdot \vec{v})\}} \left[-2B^{-1} + B^{-3} b^{-2} + 4B^{-1} (\vec{v} \cdot \vec{v}) - 2B^{-1} (\vec{v} \cdot \vec{v})^2 \right]$$

$$+ 2MB^{-2} \{1 - (\vec{v} \cdot \vec{v})\} \mathcal{R}^3$$

$$- \frac{MB^{-4} b^{-2} \mathcal{R}^3}{\{1 - (\vec{v} \cdot \vec{v})\}}$$

$$+ \frac{MB^{-4} b^{-2} \mathcal{R}^3}{\{1 - (\vec{v} \cdot \vec{v})\}} - \frac{MB^{-4} b^{-2} \mathcal{R}^3}{\{1 - (\vec{v} \cdot \vec{v})\}}$$

$$+ \frac{MB^{-1} \mathcal{R}^3}{\{1 - (\vec{v} \cdot \vec{v})\}} \left[-2B^{-1} \right] \{1 - (\vec{v} \cdot \vec{v})\}^2$$

$$+ 2MB^{-2} \{1 - (\vec{v} \cdot \vec{v})\} \mathcal{R}^3$$

Los términos en \vec{r} se anulan.

$$(\vec{v} \cdot \vec{a}) =$$

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$$M \mathcal{R}^2 \left[-B^{-1} b^{-4} + B^{-1} b^{-2} \{1 - (\vec{v} \cdot \vec{v})\} \right]$$

$$+ M \mathcal{R}^3 \left[\begin{aligned} & -B^{-3} r b^{-2} + B^{-3} r b^{-4} + B^{-3} (\vec{r} \cdot \vec{v}) b^{-2} \\ & -2B^{-1} (\vec{r} \cdot \vec{v}) b^{-2} \{1 - (\vec{v} \cdot \vec{v})\} \\ & + 2B^{-1} (\vec{v} \cdot \vec{v}) r b^{-2} \{1 - (\vec{v} \cdot \vec{v})\} \end{aligned} \right]$$

$$+ \frac{B^{-1} (\vec{v} \cdot \vec{a})}{\{1 - (\vec{v} \cdot \vec{v})\}^2} = \underline{\underline{0/0}}$$

$$M \mathcal{R}^2 \left[-\frac{B^{-2} b^{-4}}{\{1 - (\vec{v} \cdot \vec{v})\}^2} + \frac{B^{-2} b^{-2}}{\{1 - (\vec{v} \cdot \vec{v})\}} \right]$$

$$+ M \mathcal{R}^3 \left[\begin{aligned} & -\frac{B^{-4} r b^{-2}}{\{1 - (\vec{v} \cdot \vec{v})\}^2} + \frac{B^{-4} r b^{-4}}{\{1 - (\vec{v} \cdot \vec{v})\}^2} \\ & + \frac{B^{-4} (\vec{r} \cdot \vec{v}) b^{-2}}{\{1 - (\vec{v} \cdot \vec{v})\}^2} \\ & - \frac{2B^{-2} (\vec{r} \cdot \vec{v}) b^{-2}}{\{1 - (\vec{v} \cdot \vec{v})\}} \\ & \rightarrow \oplus \frac{2B^{-2} (\vec{v} \cdot \vec{v}) r b^{-2}}{\{1 - (\vec{v} \cdot \vec{v})\}} \end{aligned} \right]$$

$$r - \vec{V} \cdot \vec{r} = R^{-1}$$

$$\boxed{\vec{V} \cdot \vec{r} = r - R^{-1}}$$

$$1 - V^2 = B^{-2}$$

$$\boxed{V^2 = 1 - B^{-2}}$$

$$(\vec{v} \cdot \vec{a}) =$$

$$M R^3 [r - R^{-1}] [-2B^{-1} + B^{-3} b^{-2} + 4B^{-1} (\vec{v} \cdot \vec{v}) - 2B^{-1} (\vec{v} \cdot \vec{v})^2]$$

$$+ M R^2 [1 - B^{-2}] [-B^{-1} b^{-2}]$$

$$+ M R^3 [1 - B^{-2}] \left[2B^{-1} r - 2B^{-1} (r \cdot \vec{v}) - 2B^{-1} (\vec{v} \cdot \vec{v}) r \right. \\ \left. + 2B^{-1} (\vec{v} \cdot \vec{v}) (r \cdot \vec{v}) \right]$$

$$+ M R^2 [B^{-1} b^{-2} (\vec{v} \cdot \vec{v})]$$

$$+ M R^3 (\vec{v} \cdot \vec{v}) \left[-B^{-3} r b^{-2} + 2B^{-1} (r \cdot \vec{v}) - 2B^{-1} (\vec{v} \cdot \vec{v}) r \right. \\ \left. - 2B^{-1} (\vec{v} \cdot \vec{v}) (r \cdot \vec{v}) + 2B^{-1} (\vec{v} \cdot \vec{v})^2 r \right]$$

$$(\vec{v} \cdot \vec{a}) =$$

$$[M R^3 r - M R^2] [-2B^{-1} \{1 - (\vec{v} \cdot \vec{v})\}^2 + B^{-3} b^{-2}]$$

$$\rightarrow M B^{-1} b^{-2} R^2 + M B^{-3} b^{-2} R^2$$

$$+ [M R^3 - M B^{-2} R^3] \left[2B^{-1} r \{1 - (\vec{v} \cdot \vec{v})\} \right. \\ \left. - 2B^{-1} (r \cdot \vec{v}) \{1 - (\vec{v} \cdot \vec{v})\} \right]$$

$$+ M B^{-1} (\vec{v} \cdot \vec{v}) b^{-2} R^2$$

$$+ M (\vec{v} \cdot \vec{v}) R^3 \left[-B^{-3} r b^{-2} + 2B^{-1} (r \cdot \vec{v}) \{1 - (\vec{v} \cdot \vec{v})\} \right. \\ \left. - 2B^{-1} (\vec{v} \cdot \vec{v}) r \{1 - (\vec{v} \cdot \vec{v})\} \right]$$

$$(\vec{v} \cdot \vec{a}) =$$

 $\mathcal{M} \mathcal{R}^2$

$$+ 2B^{-1} \{1 - (\vec{v} \cdot \vec{v})\}^2$$

$$- B^{-3} b^{-2}$$

$$- B^{-1} b^{-2}$$

$$+ B^{-3} b^{-2}$$

$$+ B^{-1} (\vec{v} \cdot \vec{v}) b^{-2}$$

 $\mathcal{M} \mathcal{R}^3$

$$- 2B^{-1} r \{1 - (\vec{v} \cdot \vec{v})\}^2 \checkmark$$

$$+ B^{-3} r b^{-2} \checkmark$$

$$+ 2B^{-1} r \{1 - (\vec{v} \cdot \vec{v})\} \checkmark$$

$$- 2B^{-1} (\vec{r} \cdot \vec{v}) \{1 - (\vec{v} \cdot \vec{v})\} \checkmark$$

$$- 2B^{-3} r \{1 - (\vec{v} \cdot \vec{v})\} \checkmark$$

$$+ 2B^{-3} (\vec{r} \cdot \vec{v}) \{1 - (\vec{v} \cdot \vec{v})\} \checkmark$$

$$- B^{-3} (\vec{v} \cdot \vec{v}) r b^{-2} \checkmark$$

$$+ 2B^{-1} (\vec{v} \cdot \vec{v}) (\vec{r} \cdot \vec{v}) \{1 - (\vec{v} \cdot \vec{v})\} \checkmark$$

$$- 2B^{-1} (\vec{v} \cdot \vec{v})^2 r \{1 - (\vec{v} \cdot \vec{v})\} \checkmark$$

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$$(\vec{v} \cdot \vec{a}) =$$

$$\frac{MR^2}{2}$$

$$+ 2B^{-1} \{1 - (\vec{v} \cdot \vec{v})\}^2$$

$$- B^{-1} b^{-2} \{1 - (\vec{v} \cdot \vec{v})\}$$

$$\frac{MR^3}{2}$$

$$+ B^{-3} r b^{-2} \{1 - (\vec{v} \cdot \vec{v})\}$$

$$+ 2B^{-1} r \{1 - (\vec{v} \cdot \vec{v})\}$$

$$- 2B^{-1} r \{1 - (\vec{v} \cdot \vec{v})\}^2$$

$$- 2B^{-1} (\vec{v} \cdot \vec{v}) \{1 - (\vec{v} \cdot \vec{v})\}^2$$

$$- 2B^{-1} (\vec{v} \cdot \vec{v})^2 r \{1 - (\vec{v} \cdot \vec{v})\}$$

$$+ 2B^{-3} r \{1 - (\vec{v} \cdot \vec{v})\}$$

$$+ 2B^{-3} (\vec{v} \cdot \vec{v}) \{1 - (\vec{v} \cdot \vec{v})\}$$

$$- \frac{B^{-1} b^{-2} (\vec{v} \cdot \vec{a})}{\{1 - (\vec{v} \cdot \vec{v})\}^3}$$

$$\frac{M R^2}{- \frac{2 B^{-2} b^{-2}}{\{1 - (\vec{v} \cdot \vec{v})\}} + \frac{B^{-2} b^{-4}}{\{1 - (\vec{v} \cdot \vec{v})\}^2}}$$

$$\begin{aligned} & - \frac{B^{-4} M R^3}{\{1 - (\vec{v} \cdot \vec{v})\}^2} \\ & - \frac{2 B^{-2} r b^{-2}}{\{1 - (\vec{v} \cdot \vec{v})\}^2} \\ & + \frac{2 B^{-2} r b^{-2}}{\{1 - (\vec{v} \cdot \vec{v})\}^2} \\ & + \frac{2 B^{-2} (\vec{r} \cdot \vec{v}) b^{-2}}{\{1 - (\vec{v} \cdot \vec{v})\}} \\ & + \frac{2 B^{-2} (\vec{v} \cdot \vec{v})^2 r b^{-2}}{\{1 - (\vec{v} \cdot \vec{v})\}^2} \\ & + \frac{2 B^{-4} r b^{-2}}{\{1 - (\vec{v} \cdot \vec{v})\}^2} \\ & - \frac{2 B^{-4} (\vec{r} \cdot \vec{v}) b^{-2}}{\{1 - (\vec{v} \cdot \vec{v})\}^2} \end{aligned}$$

Términos en \vec{V}

$M\mathcal{R}^2$

$$-\frac{B^{-2}b^{-2}}{\{1 - (\vec{V} \cdot \vec{v})\}}$$

$$-\frac{2B^{-2}b^{-2}}{\{1 - (\vec{V} \cdot \vec{v})\}}$$

$$+\frac{B^{-2}b^{-4}}{\{1 - (\vec{V} \cdot \vec{v})\}^2}$$

$$-\frac{B^{-2}b^{-4}}{\{1 - (\vec{V} \cdot \vec{v})\}^2}$$

$$+\frac{B^{-2}b^{-2}}{\{1 - (\vec{V} \cdot \vec{v})\}}$$

$$+\frac{\cancel{B}^{-2}b^{-2}}{\{1 - (\vec{V} \cdot \vec{v})\}}$$

$$+ 2$$

$$- 2(\vec{V} \cdot \vec{v})$$

$$- 2\{1 - (\vec{V} \cdot \vec{v})\}$$

$$+\frac{B^{-2}b^{-2}}{\{1 - (\vec{V} \cdot \vec{v})\}}$$

Se anulan los términos en $M\mathcal{R}^2\vec{V}$ en el desarrollo de

$$e^{-2MB^{-1}\mathcal{R}} \left[\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \vec{v}} - \nabla \mathcal{L} \right]$$

(al sustituir el valor de \vec{a}_k)
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$$D = 1 - (\vec{V} \cdot \vec{v})$$

$$-2B^{-2} r b^{-2} D^{-2} \{1 - (\vec{V} \cdot \vec{v})\}^2$$

$$+ 2B^{-2} r b^{-2} D^{-1} \{1 + (\vec{V} \cdot \vec{v})\}$$

$$\rightarrow -2B^{-2} r b^{-2} D^{-2} \{1 - (\vec{V} \cdot \vec{v})\} \{1 + (\vec{V} \cdot \vec{v})\}$$

$$\rightarrow -2B^{-2} r b^{-2} D^{-1} \{1 + (\vec{V} \cdot \vec{v})\}$$

Se anulan los términos en $M \mathcal{R}^3 \vec{V}$ en el desarrollo de $e^{-2MB^{-1}\mathcal{R}} \left[\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \vec{v}} - \nabla \mathcal{L} \right]$ al sustituir el valor de \vec{a} (Cuaderno grueso página 214)

← ojo

MR³

~~2B⁻²r~~

~~-2B⁻²($\vec{r} \cdot \vec{v}$)~~

~~-B⁻⁴rb⁻⁴D⁻²~~

~~2B⁻²rb⁻²D⁻²~~ rule

~~2B⁻²r~~

~~+2B⁻²r~~

~~-2B⁻²($\vec{r} \cdot \vec{v}$)~~

⊕ 2B⁻²rb⁻²D⁻¹

+ 2B⁻²($\vec{r} \cdot \vec{v}$)b⁻²D⁻¹

+ 2B⁻²b⁻²($\vec{v} \cdot \vec{v}$)²rD⁻²

- 2B⁻²rb⁻²D⁻²(1 - $\vec{v} \cdot \vec{v}$)

+ 2B⁻⁴rb⁻²D⁻²

- 2B⁻⁴($\vec{r} \cdot \vec{v}$)b⁻²D⁻²

- B⁻⁴rb⁻²D⁻²

+ 2B⁻²rb⁻²D⁻²(1 + $\vec{v} \cdot \vec{v}$)

+ B⁻⁴rb⁻⁴D⁻²

+ B⁻⁴($\vec{r} \cdot \vec{v}$)b⁻²D⁻²

- 2B⁻²($\vec{r} \cdot \vec{v}$)b⁻²D⁻¹

⊕ 2B⁻²($\vec{v} \cdot \vec{v}$)rb⁻²D⁻¹

= B⁻⁴rb⁻²D⁻²

+ B⁻⁴($\vec{r} \cdot \vec{v}$)b⁻²D⁻²

- 2B⁻²r

+ 2B⁻²($\vec{r} \cdot \vec{v}$)

Términos en $M\mathcal{R}^2\vec{v}$

(2) $+ B^{-2} b^{-2} D^{-1}$
 $+ 2B^{-2}$
 $- B^{-2} b^{-2} D^{-1}$
 $- 2B^{-2} D^{-1}$
 $+ 2B^{-2} (\vec{v} \cdot \vec{v}) / D^{-1}$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \vec{v}} \right)$$

$$-2B^{-2} D^{-1} \{1 - (\vec{v} \cdot \vec{v})\} =$$

$$-2B^{-2}$$

Se anulan los términos en $M\mathcal{R}^2\vec{v}$ en el desarrollo de $e^{-2MB^{-1}R} \left[\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \vec{v}} - \nabla \mathcal{L} \right]$ al sustituir el valor de \vec{a} (Cuaderno grueso pág. 214)

Se anulan los términos
en $M \mathbb{R}^3 \vec{v}$ en el desarrollo
de

$$e^{-2MB^{-1}R} \left[\frac{d}{dt} \frac{\partial L}{\partial \vec{v}} - \nabla L \right]$$

al sustituir el valor de \vec{a}
(Cuaderno grueso pág. 214)



Términos en MR \vec{v}

$$\begin{aligned}
 & \cancel{-B^{-4} r b^{-2} D^{-1}} \\
 & + 2B^{-2} (\vec{r} \cdot \vec{v}) \\
 & - 2B^{-2} (\vec{v} \cdot \vec{v}) r \\
 & + \cancel{B^{-4} r b^{-2} D^{-1}} \\
 & + 2B^{-2} r D^{-1} \\
 & - 2B^2 r \\
 & - \cancel{2B^{-2} (\vec{r} \cdot \vec{v})} \\
 & - 2B^{-2} (\vec{v} \cdot \vec{v})^2 r D^{-1} \\
 & - \cancel{2B^{-4} r D^{-1}} \\
 & + \cancel{2B^{-4} (\vec{r} \cdot \vec{v}) D^{-1}} \\
 & + \cancel{2B^{-4} r D^{-1}} \\
 & - \cancel{2B^{-4} (\vec{r} \cdot \vec{v}) D^{-1}}
 \end{aligned}$$

$\left. \begin{matrix} \text{---} \\ \text{---} \\ \text{---} \end{matrix} \right\} \vec{a}$
 $\left. \begin{matrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{matrix} \right\} (\vec{v} \cdot \vec{a})$
 $\left. \begin{matrix} \text{---} \\ \text{---} \end{matrix} \right\} \frac{d}{dt} \left(\frac{\partial L}{\partial \vec{v}} \right)$

$$\begin{aligned}
 & - 2B^{-2} r \{1 + (\vec{v} \cdot \vec{v})\} + 2B^{-2} r D^{-1} \{1 - (\vec{v} \cdot \vec{v})^2\} \\
 & - 2B^{-2} r \{1 + (\vec{v} \cdot \vec{v})\} + 2B^{-2} r D^{-1} \{1 + (\vec{v} \cdot \vec{v})\} \{1 + (\vec{v} \cdot \vec{v})\}
 \end{aligned}$$

Las extremales correspondientes al Lagrangiano

$$\mathcal{L} = \frac{1}{2} e^{+2MB^{-1}\mathcal{R}} \left[2B(1 - (\vec{V} \cdot \vec{v})) - \frac{B^{-1}b^{-2}}{1 - (\vec{V} \cdot \vec{v})} \right]$$

en el problema variacional

$$\delta \int_{t_0}^{t_1} \mathcal{L} dt = 0$$

$$\begin{cases} \vec{V} = \text{const} \\ \vec{A} = \vec{0} \end{cases}$$

son las siguientes: $\frac{\vec{a}}{M} =$

$$\mathcal{R}^3 \left[-2B^{-1} + B^{-3}b^{-2} + 4B^{-1}(\vec{V} \cdot \vec{v}) - 2B^{-1}(\vec{V} \cdot \vec{v})^2 \right] \vec{r} + \left[\begin{aligned} &+ \mathcal{R}^2 [-B^{-1}b^{-2}] \\ &+ \mathcal{R}^3 \left[+2B^{-1}r - 2B^{-1}(\vec{r} \cdot \vec{v}) - 2B^{-1}(\vec{V} \cdot \vec{v})r \right. \\ &\quad \left. + 2B^{-1}(\vec{V} \cdot \vec{v})(\vec{r} \cdot \vec{v}) \right] \end{aligned} \right] \vec{V}$$

$$+ \left[\begin{aligned} &+ \mathcal{R}^2 [B^{-1}b^{-2}] \\ &+ \mathcal{R}^3 \left[-B^{-3}rb^{-2} + 2B^{-1}(\vec{r} \cdot \vec{v}) - 2B^{-1}(\vec{V} \cdot \vec{v})r \right. \\ &\quad \left. - 2B^{-1}(\vec{V} \cdot \vec{v})(\vec{r} \cdot \vec{v}) + 2B^{-1}(\vec{V} \cdot \vec{v})^2 r \right] \end{aligned} \right] \vec{v}$$

#

Campo probado mismo

Esta es la ecuación diferencial del movimiento de partículas exploradoras en el campo de un punto masa que se mueve con velocidad constante \vec{V} . ($\vec{A} = 0$).

$$R = \frac{1}{r - \vec{r} \cdot \vec{V}}$$

$$B = \frac{1}{\sqrt{1 - V^2}}$$

$$b = \frac{1}{\sqrt{1 - V^2}}$$

Notación!

- M --- masa en reposo del punto masa
- \vec{V} --- vector velocidad retardado del punto masa
- \vec{a} --- vector aceleración de la partícula exploradora
- \vec{v} --- vector velocidad de la partícula exploradora

#

(x, y, z) posición de la partícula exploradora en el instante t .

(X, Y, Z) posición retardada del punto masa para la posición de la partícula exploradora en (x, y, z) en el instante t .

$$r = \sqrt{(x-X)^2 + (y-Y)^2 + (z-Z)^2}$$

$$\vec{r} = (x-X, y-Y, z-Z).$$

Desarrollo en serie del Lagrangiano.

$$e^{2MB^{-1}\mathcal{R}} = 1 + 2MB^{-1}\mathcal{R} + 2M^2B^{-2}\mathcal{R}^2 + \dots$$

$$\frac{1}{\{1 - \vec{V} \cdot \vec{V}\}} = 1 + (\vec{V} \cdot \vec{V}) + (\vec{V} \cdot \vec{V})^2 + \dots$$

$$\frac{1}{2}[\] = \left[\begin{array}{l} B - B(\vec{V} \cdot \vec{V}) - \frac{1}{2}B^{-1}b^{-2} \\ - \frac{1}{2}B^{-1}b^{-2}(\vec{V} \cdot \vec{V}) - \frac{1}{2}B^{-1}b^{-2}(\vec{V} \cdot \vec{V})^2 \end{array} \right]$$

$$\frac{1}{2}[\] = \left[\begin{array}{l} (B - \frac{1}{2}B^{-1}b^{-2}) \\ - \{B + \frac{1}{2}B^{-1}b^{-2}\}(\vec{V} \cdot \vec{V}) \\ - \frac{1}{2}B^{-1}b^{-2}(\vec{V} \cdot \vec{V})^2 \end{array} \right]$$

$$B - \left(\frac{1}{2} B^{-1} b^{-2} \right) \left\{ B + \frac{1}{2} B^{-1} b^{-2} \right\} (\vec{V} \cdot \vec{V}) - \frac{1}{2}$$

L =

$$\begin{aligned}
 & \left(B - \frac{1}{2} B^{-1} b^{-2} \right) \\
 & + \left[- \left\{ B + \frac{1}{2} B^{-1} b^{-2} \right\} (\vec{V} \cdot \vec{v}) \right. \\
 & \quad \left. + 2M \left\{ 1 - \frac{1}{2} B^{-2} b^{-2} \right\} \mathcal{R} \right] \\
 & + \left[- \frac{1}{2} B^{-1} b^{-2} (\vec{V} \cdot \vec{v})^2 \right. \\
 & \quad \left. - 2M \left\{ 1 + \frac{1}{2} B^{-2} b^{-2} \right\} \mathcal{R} (\vec{V} \cdot \vec{v}) \right. \\
 & \quad \left. + 2M^2 \left\{ B^{-1} - \frac{1}{2} B^{-3} b^{-2} \right\} \mathcal{R}^2 \right]
 \end{aligned}$$

L =

$$\begin{aligned}
 & B - \frac{1}{2} B^{-1} + \frac{1}{2} B^{-1} v^2 \\
 & - B (\vec{V} \cdot \vec{v}) - \frac{1}{2} B^{-1} (\vec{V} \cdot \vec{v}) + \frac{1}{2} B^{-1} (\vec{V} \cdot \vec{v}) v^2 \\
 & + 2M \mathcal{R} - M B^{-2} \mathcal{R} + M B^{-2} \mathcal{R} v^2 \\
 & - \frac{1}{2} B^{-1} (\vec{V} \cdot \vec{v})^2 + \frac{1}{2} B^{-1} (\vec{V} \cdot \vec{v})^2 v^2 \\
 & - 2M \mathcal{R} (\vec{V} \cdot \vec{v}) - M B^{-2} (\vec{V} \cdot \vec{v}) \mathcal{R} \\
 & + M B^{-2} (\vec{V} \cdot \vec{v}) \mathcal{R} v^2 + 2M^2 B^{-1} \mathcal{R}^2 \\
 & - M^2 B^{-3} \mathcal{R}^2 + M^2 B^{-3} \mathcal{R}^2 v^2
 \end{aligned}$$

$$B - \frac{1}{2} B^{-1} + \frac{1}{2} B^{-1} v^2 = B(\vec{V} \cdot \vec{V})$$

$$\mathcal{L} =$$

$$B - \frac{1}{2} B^{-1}$$

$$+ \frac{1}{2} B^{-1} v^2 - B(\vec{V} \cdot \vec{V}) - \frac{1}{2} B^{-1} (\vec{V} \cdot \vec{V})$$

$$+ 2M\mathcal{R} - MB^{-2}\mathcal{R}$$

$$+ \frac{1}{2} B^{-1} (\vec{V} \cdot \vec{V}) v^2 + MB^{-2}\mathcal{R} v^2$$

$$- \frac{1}{2} B^{-1} (\vec{V} \cdot \vec{V})^2 - 2M\mathcal{R} (\vec{V} \cdot \vec{V})$$

$$- MB^{-2} (\vec{V} \cdot \vec{V}) \mathcal{R}$$

$$- M^2 B^{-3} \mathcal{R}^2 + \dots$$

$$+ 2M^2 B^{-1} \mathcal{R}^2$$

$$A_{\alpha\beta\gamma\delta\epsilon} M^\alpha B^{-\beta} (\vec{V} \cdot \vec{v})^\delta R_{\nu}^{\sigma}{}_{2\epsilon}$$

$$\frac{1}{1 - (\vec{v} \cdot \vec{v})} = 1 + (\vec{v} \cdot \vec{v}) + (\vec{v} \cdot \vec{v})^2 + \dots$$

$$\frac{\frac{1}{2} B^{-1} b^{-2}}{\{1 - (\vec{v} \cdot \vec{v})\}} = \left\{ \begin{aligned} &\frac{1}{2} B^{-1} b^{-2} + \frac{1}{2} B^{-1} b^{-2} (\vec{v} \cdot \vec{v}) \\ &+ \frac{1}{2} B^{-1} b^{-2} (\vec{v} \cdot \vec{v})^2 + \dots \end{aligned} \right.$$

$$b^{-2} = 1 - v^2$$

$$\frac{\frac{1}{2} B^{-1} b^{-2}}{\{1 - (\vec{v} \cdot \vec{v})\}} \approx \left[\begin{aligned} &\frac{1}{2} B^{-1} - \frac{1}{2} B^{-1} v^2 \\ &+ \frac{1}{2} B^{-1} (\vec{v} \cdot \vec{v}) - \frac{1}{2} B^{-1} (\vec{v} \cdot \vec{v})^2 \\ &+ \frac{1}{2} B^{-1} (\vec{v} \cdot \vec{v})^2 + \dots \end{aligned} \right.$$

$$\frac{1}{2} [] = \left[\begin{aligned} &B - \frac{1}{2} B^{-1} + \frac{1}{2} B^{-1} v^2 \\ &- B (\vec{v} \cdot \vec{v}) - \frac{1}{2} B^{-1} (\vec{v} \cdot \vec{v}) \\ &+ \frac{1}{2} B^{-1} (\vec{v} \cdot \vec{v}) v^2 + \frac{1}{2} B^{-1} (\vec{v} \cdot \vec{v})^2 \\ &+ \end{aligned} \right.$$

$$e^{+2M B^{-1} \mathcal{R}} = \frac{1}{2} [] =$$

1

$$B - \frac{1}{2} B^{-1}$$

$$+ 2M B^{-1} \mathcal{R}$$

$$+ \frac{1}{2} B^{-1} v^2 - B(\vec{V} \cdot \vec{v}) - \frac{1}{2} B^{-1} (\vec{V} \cdot \vec{v})$$

$$+ 2M^2 B^{-2} \mathcal{R}^2$$

$$+ \frac{1}{2} B^{-1} (\vec{V} \cdot \vec{v}) v^2 - \frac{1}{2} B^{-1} (\vec{V} \cdot \vec{v})^2$$

+ ...

+ ...

$$\mathcal{L} =$$

$$B - \frac{1}{2} B^{-1}$$

$$+ \frac{1}{2} B^{-1} v^2 - B(\vec{V} \cdot \vec{v}) - \frac{1}{2} B^{-1} (\vec{V} \cdot \vec{v})$$

$$+ 2M \mathcal{R} - M B^{-2} \mathcal{R}$$

$$+ \frac{1}{2} B^{-1} (\vec{V} \cdot \vec{v}) v^2 - \frac{1}{2} B^{-1} (\vec{V} \cdot \vec{v})^2 + \dots$$

$$+ M B^{-2} \mathcal{R} v^2 - 2M (\vec{V} \cdot \vec{v}) \mathcal{R} - M B^{-2} (\vec{V} \cdot \vec{v}) \mathcal{R}$$

$$+ 2M^2 \mathcal{R}^2 - M^2 B^{-3} \mathcal{R}^2 + \dots$$

Primeros terminos del desarrollo de \mathcal{L} .

$$\mathcal{L} = B - \frac{1}{2} B^{-1} v^2$$

$$+ \frac{1}{2} B^{-1} v^2 - B(\vec{V} \cdot \vec{v}) - \frac{1}{2} B^{-1} (\vec{V} \cdot \vec{v})^2 + 2M\mathcal{R} - MB^{-2}\mathcal{R}$$

$$+ \frac{1}{2} B^{-1} (\vec{V} \cdot \vec{v}) v^2 - \frac{1}{2} B^{-1} (\vec{V} \cdot \vec{v})^2 + MB^{-2}\mathcal{R} v^2 - 2M(\vec{V} \cdot \vec{v})\mathcal{R} - MB^{-2}(\vec{V} \cdot \vec{v})\mathcal{R} + 2M^2 B^{-1}\mathcal{R}^2 - M^2 B^{-3}\mathcal{R}^2 + \dots$$

¡Comprobado!

$$\mathcal{L} = \sum A_{\alpha\beta\gamma} \delta\epsilon M^\alpha B^{-\beta} (\vec{V} \cdot \vec{v})^\gamma \mathcal{R}^\delta \rightarrow v^{2\epsilon}$$



$$B = \sqrt{1 - v^2}$$

$$B = \frac{1}{\sqrt{1 - v^2}} \approx 1 - \frac{1}{2}(-v^2) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{1 \cdot 2}(+v^4)$$

$$B \approx 1 + \frac{1}{2}v^2 + \frac{3}{8}v^4 + \dots$$

$$(1 - v^2)^{-\frac{1}{2}} = 1 - \frac{1}{2}(-v^2) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{1 \cdot 2}(-v^2)^2$$

$$B^{-1} = \sqrt{1 - v^2} = 1 + \frac{1}{2}(-v^2) + \frac{(+\frac{1}{2})(-\frac{3}{2})}{1 \cdot 2}(-v^2)^2$$

$$B^{-1} \approx 1 - \frac{1}{2}v^2 - \frac{1}{8}v^4 + \dots$$

$$B - \frac{1}{2}B^{-1} = \frac{1}{2} + \frac{3}{4}v^2 + \frac{7}{16}v^4 + \dots$$

$$\frac{1}{2}B^{-1}v^2 = \frac{1}{2}v^2 - \frac{1}{4}v^2v^2 - \frac{1}{16}v^4v^2$$

$$B(\vec{v} \cdot \vec{v}) = -(\vec{v} \cdot \vec{v}) - \frac{1}{2}v^2(\vec{v} \cdot \vec{v})$$

$$-\frac{1}{2}B^{-1}(\vec{v} \cdot \vec{v}) = -\frac{1}{2}(\vec{v} \cdot \vec{v}) + \frac{1}{4}v^2(\vec{v} \cdot \vec{v})$$

$$-MB^{-2}\mathcal{R} \approx -M\mathcal{R} + MV^2\mathcal{R}$$

$$+\frac{1}{2}B^{-1}(\vec{V}\cdot\vec{V})v^2 \approx \frac{1}{2}(\vec{V}\cdot\vec{V})v^2 + \dots$$

$$-\frac{1}{2}B^{-1}(\vec{V}\cdot\vec{V})^2 \approx -\frac{1}{2}(\vec{V}\cdot\vec{V})^2 + \dots$$

$$+MB^{-2}v^2\mathcal{R} \approx Mv^2\mathcal{R} + \dots$$

$$-2M(\vec{V}\cdot\vec{V})\mathcal{R} = -2M(\vec{V}\cdot\vec{V})\mathcal{R}$$

$$-MB^{-2}(\vec{V}\cdot\vec{V})\mathcal{R} = -M(\vec{V}\cdot\vec{V})\mathcal{R}$$

$$+2M^2B^{-1}\mathcal{R}^2 \approx +2M^2\mathcal{R}^2$$

$$-M^2B^{-3}\mathcal{R}^2 \approx -M^2\mathcal{R}^2$$

$$\mathcal{L} =$$

60

 $\frac{1}{2}$

$$\begin{aligned} & + \frac{3}{4} V^2 \\ & + \frac{1}{2} v^2 m \\ & - (\vec{V} \cdot \vec{v}) \\ & - \frac{1}{2} (\vec{V} \cdot \vec{v}) \\ & + 2M\mathcal{R} \\ & - M\mathcal{R} \\ & + \cancel{M(\vec{V} \cdot \vec{v})\mathcal{R}} \end{aligned}$$

$$\begin{aligned} & + \frac{7}{16} V^4 \\ & - \frac{1}{4} V^2 v^2 \\ & - \frac{1}{2} V^2 (\vec{V} \cdot \vec{v}) \\ & + \frac{1}{4} V^2 (\vec{V} \cdot \vec{v}) \\ & + Mv^2\mathcal{R} \\ & + \frac{1}{2} (\vec{V} \cdot \vec{v}) v^2 \\ & - \frac{1}{2} (\vec{V} \cdot \vec{v})^2 \\ & + Mv\mathcal{R} \\ & - 2M(\vec{V} \cdot \vec{v})\mathcal{R} \\ & - M(\vec{V} \cdot \vec{v})\mathcal{R} \\ & + 2M^2\mathcal{R}^2 \\ & - M^2\mathcal{R}^2 \end{aligned}$$

$\mathcal{L} =$

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$$\frac{1}{2} + \left[\begin{array}{l} \frac{1}{2} v^2 \\ + \frac{3}{4} V^2 \\ - \frac{3}{2} (\vec{V} \cdot \vec{v}) \\ + MR \end{array} \right] + \left[\begin{array}{l} \frac{7}{16} V^4 \\ - \frac{1}{4} V^2 v^2 \\ - \frac{1}{4} V^2 (\vec{V} \cdot \vec{v}) \\ + M V^2 \mathcal{R} \\ + \frac{1}{2} (\vec{V} \cdot \vec{v}) v^2 \\ - \frac{1}{2} (\vec{V} \cdot \vec{v})^2 \\ - 3M (\vec{V} \cdot \vec{v}) \mathcal{R} \\ + M^2 \mathcal{R}^2 \end{array} \right]$$

Com probando
pág. 64.

Comprobación de los primeros términos de L. 62

$$B \{1 - (\vec{V} \cdot \vec{v})\} = \left[1 + \left[\frac{1}{2} V^2 - \vec{V} \cdot \vec{v} \right] + \left[\frac{3}{8} V^4 - \frac{1}{2} V^2 (\vec{V} \cdot \vec{v}) \right] + \dots \right]$$

$$\frac{B^{-1}}{1 - (\vec{V} \cdot \vec{v})} = \left[1 + \left[(\vec{V} \cdot \vec{v}) - \frac{1}{2} V^2 \right] + \left[(\vec{V} \cdot \vec{v})^2 - \frac{1}{2} V^2 (\vec{V} \cdot \vec{v}) - \frac{1}{8} V^4 \right] + \dots \right]$$

$$\frac{B^{-1} b^{-2}}{1 - (\vec{V} \cdot \vec{v})} = \left[1 + \left[(\vec{V} \cdot \vec{v}) - \frac{1}{2} V^2 - v^2 \right] + \left[(\vec{V} \cdot \vec{v})^2 - \frac{1}{2} V^2 (\vec{V} \cdot \vec{v}) - \frac{1}{8} V^4 - (\vec{V} \cdot \vec{v}) v^2 + \frac{1}{2} V^2 v^2 \right] \right]$$

$$\frac{1}{2} \frac{B^{-1} b^{-2}}{1 - (\vec{V} \cdot \vec{v})} = \left[-\frac{1}{2} + \left[-\frac{1}{2} (\vec{V} \cdot \vec{v}) + \frac{1}{4} V^2 + \frac{1}{2} v^2 \right] + \left[-\frac{1}{2} (\vec{V} \cdot \vec{v})^2 + \frac{1}{4} V^2 (\vec{V} \cdot \vec{v}) + \frac{1}{16} V^4 + \frac{1}{2} (\vec{V} \cdot \vec{v}) v^2 - \frac{1}{4} V^2 v^2 \right] \right]$$

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$$\left[B(1 - (\vec{V} \cdot \vec{v})) - \frac{1}{2} \frac{B^{-1} b^{-2}}{1 - (\vec{V} \cdot \vec{v})} \right] =$$

$$\frac{1}{2} + \left[\frac{3}{4} V^{-2} - \frac{3}{2} (\vec{V} \cdot \vec{v}) + \frac{1}{2} v^2 \right]$$

$$+ \left[\frac{7}{16} V^4 - \frac{1}{4} V^2 (\vec{V} \cdot \vec{v}) - \frac{1}{2} (\vec{V} \cdot \vec{v})^2 \right. \\ \left. + \frac{1}{2} (\vec{V} \cdot \vec{v}) v^2 - \frac{1}{4} V^2 v^2 \right]$$

$$e^{2MB^{-1}\mathcal{R}} = 1 + 2MB^{-1}\mathcal{R} + 2M^2B^{-2}\mathcal{R}^2$$

$$e^{2MB^{-1}\mathcal{R}} = 1 + 2M\mathcal{R} - MV^2\mathcal{R} + 2M^2\mathcal{R}^2$$

$$\mathcal{L} =$$

$$\frac{1}{2} +$$

$$\left|
 \begin{array}{l}
 + \frac{3}{4} V^2 \\
 - \frac{3}{2} (\vec{V} \cdot \vec{v}) \\
 + \frac{1}{2} v^2 \\
 + M \mathcal{R}
 \end{array}
 \right.$$

$$\begin{array}{l}
 + \frac{7}{16} V^4 \\
 - \frac{1}{4} V^2 (\vec{V} \cdot \vec{v}) \\
 - \frac{1}{2} (\vec{V} \cdot \vec{v})^2 \\
 + \frac{1}{2} (\vec{V} \cdot \vec{v}) v^2 \\
 - \frac{1}{4} V^2 v^2 \\
 + \frac{3}{2} M V^2 \mathcal{R} \\
 - 3 M (\vec{V} \cdot \vec{v}) \mathcal{R} \\
 + M v^2 \mathcal{R} \\
 - \frac{1}{2} M V^2 \mathcal{R} \\
 + M^2 \mathcal{R}^2
 \end{array}$$

Lagrangiano: $\mathcal{L} =$

$$\frac{1}{2} \left[\begin{array}{l} + \frac{1}{2} v^2 \\ + \frac{3}{4} V^2 \\ - \frac{3}{2} (\vec{V} \cdot \vec{V}) \\ + M R \end{array} \right] + \left[\begin{array}{l} + \frac{7}{16} V^4 \\ - \frac{1}{4} V^2 v^2 \\ - \frac{1}{4} V^2 (\vec{V} \cdot \vec{V}) \\ + M V^2 R \\ + \frac{1}{2} (\vec{V} \cdot \vec{V}) v^2 \\ - \frac{1}{2} (\vec{V} \cdot \vec{V})^2 \\ - 3 M (\vec{V} \cdot \vec{V}) R \\ + M^2 R^2 \\ + M v^2 R \end{array} \right]$$

Comprova de Chadeiro 58
pág. 25

Órdenes de magnitud.

Magnitud fundamental c

$$[v] = c^{-1}$$

$$[V] = c^{-1}$$

$$[a] = c^0$$

$$[\vec{v} \cdot \vec{v}] = c^{-2}$$

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

$$ds = c \sqrt{1 - v^2} dt$$

$$[ds] = c^1$$

$$dt = c d\tau \quad \left. \begin{array}{l} \text{tiempo medido} \\ \text{en segundos} \end{array} \right\}$$

$$[dt] = c^1$$

$$\left[\frac{dx}{dt} \right] = c^{-1}$$

$$[a] = c^{-2}$$

$$a = \frac{M}{r^2}$$

$$[a] = [M] = c^{-2}$$

Expresar la aceleración \vec{a} de una ~~partícula~~ ^{partícula exploradora} ~~masa~~ en el campo de un punto masa en movimiento arbitrario hasta términos de ~~segundo~~ ^{cuarto} orden c^{-2}
 pág. 204, 205, 206, 207.

$$\vec{a} =$$

$$M \mathcal{R}^3 \left[\begin{array}{l} -2B^T - 2B(\vec{A} \cdot \vec{r}) + B^{-3} b^{-2} \\ + B^{-1}(\vec{A} \cdot \vec{r}) b^{-2} + 4B^T(\vec{V} \cdot \vec{V}) \end{array} \right] \vec{r}$$

$$+ \left[\begin{array}{l} M \mathcal{R}^2 [-B^T b^{-2}] \\ + M \mathcal{R}^3 \left[\begin{array}{l} + 2B(\vec{A} \cdot \vec{r}) r \\ + 2B^T r - 2B^T(\vec{r} \cdot \vec{V}) \end{array} \right] \end{array} \right] \vec{V}$$

$$+ M \mathcal{R}^2 2B r \vec{A}$$

$$+ \left[\begin{array}{l} M \mathcal{R}^2 [+B^{-1} b^{-2}] \\ + M \mathcal{R}^3 [-B^{-3} r b^{-2} + 2B^{-1}(\vec{r} \cdot \vec{V})] \end{array} \right] \vec{V}$$

$$\vec{a} =$$

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$$M R^3 \left[-2 \left\{ 1 - \frac{1}{2} v^2 \right\} - 2 (\vec{A} \cdot \vec{r}) + \left\{ 1 - \frac{3}{2} v^2 \right\} (1 - v^2) \right] \vec{r} \\ + (\vec{A} \cdot \vec{r}) + 4 (\vec{v} \cdot \vec{v})$$

$$+ \left[M R^2 \left[- \left\{ 1 - \frac{1}{2} v^2 \right\} (1 - v^2) \right] \right. \\ \left. + M R^3 \left[2r - 2(\vec{r} \cdot \vec{v}) \right] \right] \vec{v}$$

$$+ 2M R^2 r \vec{A}$$

$$+ \left[M R^2 \left[\cancel{\left\{ 1 - \frac{1}{2} v^2 \right\} (1 - v^2)} \right] \right. \\ \left. + M R^3 \left[-r + 2(\vec{r} \cdot \vec{v}) \right] \right] \vec{v}$$

$$\vec{a} =$$

$$M R^3 \left[-1 - \frac{1}{2} V^2 - v^2 + 4(\vec{V} \cdot \vec{v}) - (\vec{A} \cdot \vec{r}) \right] \vec{r}$$

$$+ \left[M R^2 [-1] + 2 M R^3 [r - (\vec{r} \cdot \vec{v})] \right] \vec{v}$$

$$+ 2 M R^2 r \vec{A}$$

$$+ \left[M R^2 + M R^3 [-r + 2(\vec{r} \cdot \vec{v})] \right] \vec{v}$$

$$\vec{a} =$$

$$M R^3 \left[-1 - \frac{1}{2} V^2 - v^2 + 4(\vec{V} \cdot \vec{v}) - (\vec{A} \cdot \vec{r}) \right] \vec{r}$$

$$+ \left[\begin{array}{l} -M R^2 \\ + 2M R^3 [r - (\vec{r} \cdot \vec{v})] \end{array} \right] \vec{v}$$

comproba
comproba

$$+ 2M R^2 r \vec{A}$$

comproba

$$+ \left[\begin{array}{l} +M R^2 \\ +M R^3 [r + 2(\vec{r} \cdot \vec{v})] \end{array} \right] \vec{v}$$

comproba

Con el Lagrangiano L de la página 61

$$\frac{\partial L}{\partial \vec{v}} = \left[\begin{aligned} &\vec{v} - \frac{3}{2} \vec{V} - \frac{1}{2} V^2 \vec{v} - \frac{1}{4} V^2 \vec{V} \\ &+ (\vec{V} \cdot \vec{v}) \vec{v} + \frac{1}{2} V^2 \vec{V} - (\vec{V} \cdot \vec{v}) \vec{V} \\ &- \cancel{\frac{3}{M}} 3M R \vec{V} + 4MR \vec{v} \end{aligned} \right]$$

Comprobado

$$\frac{\partial L}{\partial \vec{v}} = \left[\begin{aligned} &\left[1 - \frac{1}{2} V^2 + (\vec{V} \cdot \vec{v}) \right] \vec{v} \\ &+ \left[-\frac{3}{2} - \frac{1}{4} V^2 + \frac{1}{2} V^2 - (\vec{V} \cdot \vec{v}) - 3MR \right] \vec{V} \end{aligned} \right]$$

$$\nabla V^2$$

$$1 - V^2 = B^{-2}$$

$$V^2 = 1 - B^{-2}$$

$$\nabla V^2 = -(-2) B^{-3} \nabla B$$

$$\nabla V^2 = +2 B^{-3} \left[- \frac{B^3 (\vec{v} \cdot \vec{A})}{(r - \vec{r} \cdot \vec{v})} \right] \vec{r}$$

$$\nabla V^2 = 2 (\vec{v} \cdot \vec{A}) \nabla T$$

$$\nabla V^2 = - \frac{2 (\vec{v} \cdot \vec{A})}{\{r - (\vec{r} \cdot \vec{v})\}} \vec{r}$$

$$1) \nabla V^2 = - 2 (\vec{v} \cdot \vec{A}) \mathcal{R} \vec{r}$$

$$2) \nabla (\vec{v} \cdot \vec{v}) = - \frac{(\vec{A} \cdot \vec{v})}{\{r - (\vec{r} \cdot \vec{v})\}} \vec{r}$$

$$2) \nabla (\vec{v} \cdot \vec{v}) = - (\vec{A} \cdot \vec{v}) \mathcal{R} \vec{r}$$

3) $\nabla \mathcal{R}$

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$$(3) \nabla \mathcal{R} = -\beta^{-2} \mathcal{R}^3 \vec{r} - (\vec{A} \cdot \vec{r}) \mathcal{R}^3 \vec{r} + \mathcal{R}^2 \vec{v}$$

$$4) \nabla V^4 = \nabla (V^2)^2 = 2V^2 \nabla V^2$$

$$(4) \nabla V^4 = -4V^2 (\vec{v} \cdot \vec{A}) \mathcal{R} \vec{r}$$

$$5) \nabla \{V^2 (\vec{v} \cdot \vec{v})\} =$$

$$-V^2 (\vec{A} \cdot \vec{r}) \mathcal{R} \vec{r} - 2 (\vec{v} \cdot \vec{v}) (\vec{v} \cdot \vec{A}) \mathcal{R} \vec{r}$$

$$5) \nabla \{V^2 (\vec{v} \cdot \vec{v})\} = -V^2 (\vec{A} \cdot \vec{r}) \mathcal{R} \vec{r} - 2 (\vec{v} \cdot \vec{A}) (\vec{v} \cdot \vec{v}) \mathcal{R} \vec{r}$$

$$6) \nabla (\vec{v} \cdot \vec{v})^2 = -2 (\vec{v} \cdot \vec{v}) (\vec{A} \cdot \vec{v}) \mathcal{R} \vec{r}$$

$$\begin{aligned}
 7) \nabla_{\vec{r}} \{ (\vec{V} \cdot \vec{v}) \mathcal{R} \} &= -B^{-2} (\vec{V} \cdot \vec{v}) \mathcal{R}^3 \vec{r} \checkmark \\
 &- (\vec{V} \cdot \vec{v}) (\vec{A} \cdot \vec{r}) \mathcal{R}^3 \vec{r} \checkmark \\
 &+ (\vec{V} \cdot \vec{v}) \mathcal{R}^2 \vec{V} \checkmark \\
 &- (\vec{A} \cdot \vec{v}) \mathcal{R}^2 \vec{r} \checkmark
 \end{aligned}$$

$$\begin{aligned}
 8) \nabla \mathcal{R}^2 &= -2B^{-2} \mathcal{R}^4 \vec{r} - 2(\vec{A} \cdot \vec{r}) \mathcal{R}^4 \vec{r} \\
 &+ 2 \mathcal{R}^3 \vec{V}
 \end{aligned}$$

$$I(1) \underline{(\vec{v} \cdot \nabla) V^2}$$

I(1)

$$(\vec{v} \cdot \nabla) V^2 = -2(\vec{V} \cdot \vec{A})(\vec{r} \cdot \vec{v}) \mathcal{R}$$

$$II(1) (\vec{v} \cdot \nabla) (\vec{V} \cdot \vec{v}) = -(\vec{A} \cdot \vec{v})(\vec{r} \cdot \vec{v}) \mathcal{R}$$

$$\begin{aligned}
 III(1) (\vec{v} \cdot \nabla) \mathcal{R} &= -B^{-2} \mathcal{R}^3 (\vec{r} \cdot \vec{v}) - (\vec{A} \cdot \vec{r})(\vec{r} \cdot \vec{v}) \mathcal{R}^3 \\
 &+ \mathcal{R} (\vec{V} \cdot \vec{v}) \mathcal{R}^2
 \end{aligned}$$

$$\text{VI(1)} \quad (\vec{\nabla} \cdot \vec{\nabla}) \vec{V} = -\vec{A} \frac{(\vec{r} \cdot \vec{\nabla})}{\{r - (\vec{r} \cdot \vec{\nabla})\}}$$

$$\text{VI(1)} \quad (\vec{\nabla} \cdot \vec{\nabla}) \vec{V} = -(\vec{r} \cdot \vec{\nabla}) \mathcal{R} \vec{A}$$

$$\text{VI(2)} \quad \frac{\partial}{\partial t} \vec{V} = r \mathcal{R} \vec{A}$$

$$\text{II(2)} \quad \frac{\partial}{\partial t} (\vec{V} \cdot \vec{\nabla}) = r \mathcal{R} (\vec{A} \cdot \vec{\nabla})$$

$$\text{IV(2)} \quad \frac{\partial}{\partial t} \mathcal{R} = \left[\begin{array}{l} (\vec{V} \cdot \vec{r}) \mathcal{R}^3 \\ + (\vec{A} \cdot \vec{r}) r \mathcal{R}^3 \\ + B^{-2} r \mathcal{R}^3 \\ - r \mathcal{R}^3 \end{array} \right]$$

$$\text{V(2)} \quad \frac{\partial}{\partial t} V^2 = 2(\vec{V} \cdot \vec{A}) r \mathcal{R}$$

$$\text{I. } \frac{d}{dt} v^2 = 2(\vec{v} \cdot \vec{a}) r \mathcal{R} - 2(\vec{v} \cdot \vec{a})(\vec{r} \cdot \vec{v}) \mathcal{R} \quad \text{Compr.}$$

$$\text{II. } \frac{d}{dt} (\vec{v} \cdot \vec{v}) = \left[\begin{array}{l} -(\vec{a} \cdot \vec{v})(\vec{r} \cdot \vec{v}) \mathcal{R} \\ + (\vec{a} \cdot \vec{v}) r \mathcal{R} \\ + (\vec{v} \cdot \vec{a}) \end{array} \right] \quad \text{Compr. probado}$$

$$\text{IV. } \frac{d}{dt} \mathcal{R} = \left[\begin{array}{l} (\vec{v} \cdot \vec{r}) \mathcal{R}^3 \\ + (\vec{a} \cdot \vec{r}) r \mathcal{R}^3 \\ + B^{-2} r \mathcal{R}^3 \\ - r \mathcal{R}^3 \\ - B^{-2} (\vec{r} \cdot \vec{v}) \mathcal{R}^3 \\ - (\vec{a} \cdot \vec{r})(\vec{r} \cdot \vec{v}) \mathcal{R}^3 \\ + (\vec{v} \cdot \vec{v}) \mathcal{R}^2 \end{array} \right] \quad \text{Compr. probado}$$

$\left. \begin{array}{l} -v^2 r \mathcal{R}^3 \\ -(\vec{r} \cdot \vec{v}) \mathcal{R}^3 \\ +v^2 (\vec{r} \cdot \vec{v}) \mathcal{R}^3 \end{array} \right\}$

$$\underline{\text{VI}} \cdot \frac{d\vec{V}}{dt} = \left[\begin{array}{l} -(\vec{r} \cdot \vec{v}) \mathcal{R} \vec{A} \\ + r \mathcal{R} \vec{A} \\ - \end{array} \right]$$

Comprobado

$(\vec{V} \cdot \vec{a})$ hasta términos de 4 orden

$$\begin{aligned} (\vec{V} \cdot \vec{a}) = & \left[\begin{array}{l} -M(\vec{V} \cdot \vec{r}) \mathcal{R}^3 - M V^2 \mathcal{R}^2 \\ + 2 M V^2 \mathcal{R}^3 r \\ + \cancel{M(\vec{V} \cdot \vec{v}) \mathcal{R}^2} \\ + M(\vec{V} \cdot \vec{v}) \mathcal{R}^2 \\ + M(\vec{V} \cdot \vec{v}) \mathcal{R}^3 r \end{array} \right] \end{aligned}$$

Comprobado

$(\vec{v} \cdot \vec{a})$ hasta términos
de 4. orden.

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$$-M(\vec{A} \cdot \vec{r})R^3 - M(\vec{V} \cdot \vec{v})R^2$$

$$+ 2M(\vec{V} \cdot \vec{v})rR^3$$

$$+ Mv^2R^2$$

$$- Mv^2rR^3$$

Com probado

$$\frac{d}{dt}(\vec{V} \cdot \vec{v}) = \left[\begin{aligned} & -M(\vec{V} \cdot \vec{r})R^3 - Mv^2R^2 \\ & + 2Mv^2rR^3 + M(\vec{V} \cdot \vec{v})R^2 \\ & - M(\vec{V} \cdot \vec{v})rR^3 \\ & + (\vec{A} \cdot \vec{v})rR - (\vec{A} \cdot \vec{v})(\vec{r} \cdot \vec{v})R \end{aligned} \right]$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \vec{v}} \right) =$$

$$\left[\begin{aligned} & -(\vec{v} \cdot \vec{A}) r \mathcal{R} + (\vec{v} \cdot \vec{A})(\vec{r} \cdot \vec{v}) \mathcal{R} \\ & -M(\vec{v} \cdot \vec{r}) \mathcal{R}^3 - Mv^2 \mathcal{R}^2 \\ & + 2Mv^2 r \mathcal{R}^3 + M(\vec{v} \cdot \vec{v}) \mathcal{R}^2 \\ & -M(\vec{v} \cdot \vec{v}) r \mathcal{R}^3 + (\vec{A} \cdot \vec{v}) r \mathcal{R} \\ & -(\vec{A} \cdot \vec{v})(\vec{r} \cdot \vec{v}) \mathcal{R} \end{aligned} \right] \vec{v}$$

$$+ \begin{bmatrix} 1 \\ -\frac{1}{2} v^2 \\ +(\vec{v} \cdot \vec{v}) \end{bmatrix} \cdot \left[\begin{aligned} & M \mathcal{R}^3 \begin{bmatrix} -1 - \frac{1}{2} v^2 - v^2 \\ +4(\vec{v} \cdot \vec{v}) \\ -(\vec{A} \cdot \vec{r}) \end{bmatrix} \vec{r} \\ & + \begin{bmatrix} -M \mathcal{R}^2 \\ +2M \mathcal{R}^3 (r - (\vec{r} \cdot \vec{v})) \end{bmatrix} \vec{v} \\ & + 2M \mathcal{R}^2 r \vec{A} \\ & + \begin{bmatrix} M \mathcal{R}^2 \\ +M \mathcal{R}^3 [-r + 2(\vec{r} \cdot \vec{v})] \end{bmatrix} \vec{v} \end{aligned} \right]$$



$$\begin{aligned}
 & + \left[\begin{aligned}
 & -\frac{1}{2}(\vec{V} \cdot \vec{A}) r \mathcal{R} + \frac{1}{2}(\vec{V} \cdot \vec{A})(\vec{r} \cdot \vec{v}) \mathcal{R} \\
 & -M(\vec{r} \cdot \vec{v}) \mathcal{R}^3 - M(\vec{V} \cdot \vec{v}) \mathcal{R}^2 \\
 & + 2M(\vec{V} \cdot \vec{v}) r \mathcal{R}^3 + Mv^2 \mathcal{R}^2 - Mv^2 r \mathcal{R} \\
 & + M(\vec{V} \cdot \vec{r}) \mathcal{R}^3 + MV^2 \mathcal{R}^2 - 2MV_r \mathcal{R}^3 \\
 & - M(\vec{V} \cdot \vec{v}) \mathcal{R}^2 + M(\vec{V} \cdot \vec{v}) r \mathcal{R}^3 - (\vec{A} \cdot \vec{v}) r \mathcal{R} \\
 & + (\vec{A} \cdot \vec{v})(\vec{r} \cdot \vec{v}) \mathcal{R} - 3M(\vec{V} \cdot \vec{r}) \mathcal{R}^3 \\
 & - 3M(\vec{A} \cdot \vec{r}) r \mathcal{R}^3 + 3MV_r \mathcal{R}^3 \\
 & + 3M(\vec{r} \cdot \vec{v}) \mathcal{R}^3 - 3MV^2(\vec{r} \cdot \vec{v}) \mathcal{R}^3 \\
 & + 3M(\vec{A} \cdot \vec{r})(\vec{r} \cdot \vec{v}) \mathcal{R}^3 - 3M(\vec{V} \cdot \vec{v}) \mathcal{R}^2
 \end{aligned} \right] \vec{v}
 \end{aligned}$$

$$\begin{aligned}
 & + \left[\begin{aligned}
 & -\frac{3}{2} \\
 & -\frac{1}{4}V^2 \\
 & +\frac{1}{2}v^2 \\
 & -(\vec{V} \cdot \vec{v}) \\
 & -3M\mathcal{R}
 \end{aligned} \right] \cdot \left[\begin{aligned}
 & [r \mathcal{R} - (\vec{r} \cdot \vec{v}) \mathcal{R}] \vec{A}
 \end{aligned} \right]
 \end{aligned}$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \vec{v}} \right) =$$

hasta el
cuarto grado en c^{-1} 82

$$\left[\begin{array}{l} -M R^3 - M v^2 R^2 + 3M(\vec{V} \cdot \vec{v}) R^3 \\ -M(\vec{A} \cdot \vec{r}) R^3 - 2M^2 R^4 \end{array} \right] \vec{r}$$

$$+ \left[\begin{array}{l} -(\vec{V} \cdot \vec{A}) r R + M(\vec{V} \cdot \vec{r}) R^3 \\ + (\vec{A} \cdot \vec{v}) r R + M R^2 - M r R^3 \\ + 2M(\vec{r} \cdot \vec{v}) R^3 \end{array} \right] \vec{v}$$

$$+ \left[\begin{array}{l} -\frac{1}{2}(\vec{V} \cdot \vec{A}) r R + 2M(\vec{r} \cdot \vec{v}) R^3 \\ - 2M(\vec{V} \cdot \vec{r}) R^3 - (\vec{A} \cdot \vec{v}) r R \\ - M R^2 + 2M r R^3 - 2M(\vec{r} \cdot \vec{v}) R^3 \end{array} \right] \vec{V}$$

$$+ \left[\begin{array}{l} -\frac{3}{2} r R + \frac{3}{2}(\vec{r} \cdot \vec{v}) R \\ -\frac{1}{4} V^2 r R + \frac{1}{2} r v^2 R \\ -(\vec{V} \cdot \vec{v}) r R - M r R^2 \end{array} \right] \vec{A}$$

¡Comprobado!
Cuaderno 109.

$$-\nabla \mathcal{L} =$$

$$+ \frac{3}{2} (\vec{V} \cdot \vec{A}) \mathcal{R} \vec{r} \checkmark$$

$$- \frac{3}{2} (\vec{A} \cdot \vec{v}) \mathcal{R} \vec{r} \checkmark$$

$$+ M \mathcal{R}^3 \vec{r} - M V^2 \mathcal{R}^3 \vec{r} + M (\vec{A} \cdot \vec{r}) \mathcal{R}^3 \vec{r} \checkmark$$

$$- M \mathcal{R}^2 \vec{v} \checkmark$$

~~$$+ \frac{7}{4} V^2 (\vec{V} \cdot \vec{A}) \mathcal{R} \vec{r}$$~~

~~$$- \frac{1}{2} V^2 (\vec{V} \cdot \vec{A}) \mathcal{R} \vec{r}$$~~

~~$$- \frac{1}{2} (\vec{V} \cdot \vec{A}) (\vec{V} \cdot \vec{v}) \mathcal{R} \vec{r} - \frac{1}{4} V^2 (\vec{A} \cdot \vec{v}) \mathcal{R} \vec{r}$$~~

~~$$+ 2M (\vec{V} \cdot \vec{A}) \mathcal{R}^2 \vec{r} + M V^2 \mathcal{R}^3 \vec{r} \checkmark$$~~

~~$$- M V^4 \mathcal{R}^3 \vec{r} + M V^2 (\vec{A} \cdot \vec{r}) \mathcal{R}^3 \vec{r}$$~~

~~$$- M V^2 \mathcal{R}^2 \vec{v}$$~~

~~$$+ \frac{1}{2} (\vec{A} \cdot \vec{v}) V^2 \mathcal{R} \vec{r}$$~~

~~$$- (\vec{V} \cdot \vec{v}) (\vec{A} \cdot \vec{v}) \mathcal{R} \vec{r}$$~~

~~$$- 3M (\vec{A} \cdot \vec{v}) \mathcal{R}^2 \vec{r} - 3M (\vec{V} \cdot \vec{v}) \mathcal{R}^3 \vec{r} \checkmark$$~~

~~$$+ 3M V^2 (\vec{V} \cdot \vec{v}) \mathcal{R}^3 \vec{r} - 3M (\vec{V} \cdot \vec{v}) (\vec{A} \cdot \vec{r}) \mathcal{R}^3 \vec{r}$$~~

~~$$+ 3M (\vec{V} \cdot \vec{v}) \mathcal{R}^2 \vec{v}$$~~

← ← *siqve pag 82*

$$-\nabla \mathcal{L} =$$

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$$\left[\begin{array}{l} +\frac{3}{2}(\vec{V} \cdot \vec{A})\mathcal{R} - \frac{3}{2}(\vec{A} \cdot \vec{v})\mathcal{R} + M\mathcal{R}^3 \\ -Mv^2\mathcal{R}^3 + M(\vec{A} \cdot \vec{r})\mathcal{R}^3 + M\vec{V}^2\mathcal{R}^3 \\ -3M(\vec{V} \cdot \vec{v})\mathcal{R}^3 + 2M^2\mathcal{R}^4 \end{array} \right] \vec{r}$$
$$+ \left[-M\mathcal{R}^2 \right] \vec{V}$$